THE MATHEMATICS TEACHER

Volume XXXVI Number 2

Edited by William David Reeve

The Next Step Forward*

By F. Lynwood Wren
George Peabody College for Teachers

The great adventure which we call *Life* is a strange mixture of many peculiar ingredients. There is a great deal of comedy and much of tragedy in store for all of us. One of the greatest tragedies that can take place in the life of any individual is the betrayal of friends or loved ones. Overcome by such a tragedy, the bachelor philosopher Marcus Ordeyne was in a very hypochondriacal state of mind when, on the evening of November 12th, he turned to his diary to make the entry for the day. He wrote:

I will confess my madness. This book is a record of my morals—the finished version of the farce the high gods have called on me to play. I thought last night the curtain was rung down. I was wrong. Listen, and laugh as I do—if you can.

I fixed myself to work today. After all, I am not an idler. I earn my right to live. When I publish my History the world will be the richer by something, poor though it may be. I vow I have been more greatly, more nobly employed of late years, than I was when I earned my living at school-slavery teaching to children the most useless, the most disastrous, the most soulcramping branch of knowledge wherewith pedagogues in their insensate folly have crippled the minds and blasted the lives of thousands of their fellow-creatures—elementary mathematics. There is no more reason for any human being on God's earth to be acquainted with the Binomial Theorem or the Solution of Triangles—unless he is a professional scientist, when he can begin to specialize in mathematics at the same age as the lawyer begins to specialize in law or the surgeon in anatomy—than for him to be an expert in Choctaw, the Cabala or the Book of Mormon. I look back with feelings of shame and degradation to the days when, for the sake of a crust of bread, I prostituted my intelligence to wasting the precious hours of impressionable childhood, which could have been filled with so many beautiful and meaningful things, over this utterly futile and inhuman subject. It trains the mind—it teaches boys to think, they say. It doesn't. In reality it is a cut and dried subject easy to fit into a school curriculum. Its sacrosanctity saves educationalists an enormous amount of trouble, and its chief use is to enable mindless young men from the universities to make a dishonest living by teaching it to others, who in their turn may teach it to a future generation.[1]

This diatribe against mathematics was written more than a quarter of a century ago during moments of bitter despondency by one whose world had suddenly collapsed about him. Yet in these words many of us recognize familiar sounding expressions or sense implications which are quite modern in their import. Only a few days ago a passing pleasantry to a colleague concerning some of the workings of Elementary Education brought forth a series of pointed questions and remarks which might be summarized in three questions: "Why in heaven's name should

* Delivered on March 6, 1942 in Jackson, Miss., before a joint meeting of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.

[1] William J. Locke: *The Morals of Marcus Ordeyne.* New York: Grosset and Dunlap, 1906, pp. 244–245.

plane geometry, as now organized, have any place whatsoever in the school curriculum? Is there something sacred about the five books of plane geometry and their rut of sequential arrangement which has been worn deep through the cooperative efforts of texts and teachers to persist in the tradition of teaching that which has been taught? Why do not mathematicians and teachers of mathematics become concerned about the welfare and interests of the children in our schools to the extent that they make some effort to cast aside the rigid pattern of conventional subject matter and shape a flexible mathematical program embedded in the challenging issues of modern events?" A conversation with another colleague only a day or two later brought forth this statement: "Honestly, I can hardly see how a youngster should be expected to have a great deal of interest in 'the square on the hypotenuse' now that Singapore has fallen."

These are not localized invectives against mathematics by disgruntled individuals with professional sore thumbs. They are very definitely straws which should indicate to us which way the educational wind is inclined to blow. It is true that at the moment mathematics is receiving a great deal of renewed emphasis because of the war program. The mathematical shortcomings of the graduates of our schools and colleges are being brought out in a very sharp manner. Many young men, and their parents, are making frantic appeals for mathematical training they should have had earlier. Criticism is being rightly directed against guidance programs in the schools which, in their ignorance, have overlooked the importance of mathematics as an effective educational medium. All of this will undoubtedly serve to emphasize that mathematics is more than mere theoretical manipulation of formulas and theorems; that it has far-reaching values which are highly practical and functional. Let us not fool ourselves however, into thinking that mathematics has been restored to its permanent place of importance in the educational sun and that our professional worries are all over. Let us not forget that mathematics was needed in the last war and in previous wars. Since 1918 has been the period of severest criticism of the teaching of mathematics in our schools. All of the criticism, however, has not been directed against mathematics alone. In the words of Professor J. L. Mursell:

> We go through a set of complicated and costly motions called 'teaching' algebra, geometry, physics, chemistry, history, English, Latin, and so forth. And what happens? Almost nothing at all! This may seem incredible. But it is true. The subjects are 'taught.' But they are not being learned.[2]

These problems have not gone unnoticed. Committees and individuals have been studying them from many different angles. To teachers of mathematics the four most important studies are probably the Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics,[3] the Report of the National Council Committee on Arithmetic,[4] the Report of the Committee on the Function of Mathematics in General Education,[5] and the Report of the War-Preparedness Committee of the American Mathematical Society and the Mathematical Association of America.[6] Another report, not primarily on mathematics, but one in

[2] J. L. Mursell, "The Reform of the Schools," *The Atlantic Monthly* (CLXIV, 1939), 803.

[3] Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, *The Place of Mathematics in Secondary Education.* (New York: Bureau of Publications Teachers College, Columbia University, 1940.)

[4] National Council Committee on Arithmetic, "Arithmetic in General Education," *Sixteenth Yearbook of the National Council of Teachers of Mathematics.* (New York: Bureau of Publications, Teachers College, Columbia University, 1941.)

[5] Committee on the Function of Mathematics in General Education, *Mathematics in General Education.* (New York: D. Appleton-Century Company, 1940.)

[6] Marston Morse and William L. Hart, "Mathematics in the Defense Program," THE

which all teachers of mathematics should have an interest, is that known as *The Story of the Eight-Year Study*.[7] This is a study of the progress of 1,475 graduates of thirty high schools of the progressive type as they were contrasted with comparable graduates of schools of the conventional type. Of most immediate interest is the following comment on this report taken from the columns of a recent issue of *Time:*

> Progressive Education has passed its high-school test. An unofficial supreme court of educators—found that graduates of Progressive preparatory schools were more successful in college than their contemporaries. Many considered its report a death sentence for the traditional system of United States high-school education. . . . The judges found that the Progressive students got slightly better marks from their professors; won more academic honors; were more precise and systematic in their thinking, more resourceful in meeting practical problems; read more books; did more dancing; went to more concerts; took a keener interest in world affairs; went out for more extracurricular activities; were elected to more student offices. Clinching fact: graduates of the six most progressive schools had the best records, the biggest margin of superiority over their fellows. . . . In the light of these findings, *The Story of the Eight-Year Study* may outline the high-school pattern of the future.[8]

What are the implications from the above considerations for teachers of mathematics? What should we do about it? Our American system of public education is posited upon the fundamental assumption that democracy is a desirable mode of living. The educational program is therefore obligated to make the functional relationship of the individual to society its basic frame of reference. This implies that the pupils in our schools should be afforded opportunities to participate in those group activities which would provide for them desirable first-hand social experiences. In order that these pupils might not stumble to ruin through lack of adequate guidance the program must also accept the responsibility for the provision of those backgrounds of information essential for the integration of thought patterns and the rationalization of behaviour standards. In much of the recent literature on curriculum construction emphasis upon this "social motif" has greatly minimized any attention to systematic development of the higher mental processes which are of such great importance to those individuals capable of them and to the race to which those individuals belong.

The new educational philosophy of today, to a very large degree, tends to use the child rather than subject matter as the fundamental matrix of educational procedure. There is danger that such a philosophy might tend to substitute haphazard learning for systematic learning and to overlook the important fact that the pattern for efficient living is factual in detail as well as functional in relief. The educator should keep well in mind that instead of being interrupted by a discontinuity of unrelated interests, the educative process can best function through a continuity of effort. During recent years there has been a great deal of controversy concerning the educational values of mathematics. Some contend that such values are restricted to the more practical or tool functions of the subject; others insist that there are cultural, propaedeutic, and other less tangible values which should receive just as much emphasis as the practical. No educational program can meet its full responsibility to the individual pupil if it merely makes provision for the acquiring of those skills and concepts found to be valuable only when interpreted in terms of immediate usefulness. The minimum program must never be permitted to become the norm. The

---

MATHEMATICS TEACHER (XXXIV, 1941), 195–202.

William L. Hart, "Progress Report of the Subcommittee on Education for Service of the War-preparedness Committee of the American Mathematical Society and the Mathematical Association of America," THE MATHEMATICS TEACHER (XXXIV, 1941), 297–304.

[7] W. M. Aikin, *The Story of the Eight-Year Study*, (New York: Harper and Brothers, 1942.)

[8] *Time* (XXXIX, February 16, 1942), 53–54.

education of the youth of our land is too large and too serious a responsibility to allow it to become dominated by a superficial philosophy of evanescent interests and transitory values.

The report on the eight-year study states that one of the prime objectives for high-school education in the United States must be an understanding of mathematics. It insists, however, that "cut-and-dried text-book teaching should give way to teaching about the problems of modern living," and that the students' general welfare must become the heart of the curriculum which should try "to bring to every young American his great heritage of freedom (and to) inspire devotion to human welfare." Teachers of mathematics must concern themselves with the content of their instructional material. They must no longer be satisfied to travel the narrow trail of specified text-book content; a trail worn too smooth already by the weary trodding of the poorly prepared and uninspired individuals who must follow the traditional path lest they trip over unexpected questions from inquisitive upstarts, better known as bright students who really would like to do some individual thinking if given a chance. The teacher of mathematics is indeed a cultural salesman. "He has a glorious bill of goods—the noble science of mathematics. He has a glorious public—young minds which need enlightenment and which are sure to respond eagerly to a real thing if only they can get it. How shall he make mathematics real, vital, helpful to them, as it can and ought to be? This is his central problem."[9] No teacher of mathematics is deserving of the honor of the title unless he is alive to the world of number, form, and size in which he lives and finds his being. We are submerged every day and in any community, no matter how humble or how grand, in a world that is characteristically mathematical in nature. It is the unrelenting responsibility of every teacher of mathematics worthy of the name to be alive to his mathematical environment and to enrich the lives of his pupils by bringing them into the atmosphere in which he is privileged to live. May the teachers of mathematics no longer be misers of mathematical knowledge nor spendthrifts of students' interests. May they constantly be mindful of the fact that there is more challenge and thrill in the sparkle of new specie freshly turned from the mint of human experience than in the worn bronze coinage of treasured tradition.

The three great instruments for the progress of mankind are (1) *research*, which is the source for discovery of new knowledge; (2) *education*, which is the avenue through which each generation may pass on to the next its fund of accumulated knowledge; and (3) *service*, which is the opportunity for the application or modification of any fragment of acquired knowledge. The specialist in mathematics is the producer of mathematical knowledge. He may glory in the mere beauty of mathematical composition or the joy of mathematical achievement. The teacher of mathematics is the one who is to be primarily the seller of mathematics. In addition to possessing that scholarship essential to the satisfactory fulfillment of his professional obligation, the teacher of mathematics should be familiar with and know how to make use of efficient techniques of instruction. We do not want the teacher of mathematics to be a teacher who has nothing to teach, neither do we want him to be a mere purveyor of knowledge or a promoter of skill. He is the one who has the responsibility of developing interest in mathematics among the pupils in our schools and among the lay public. The user of mathematics is the one whose privilege it is to evaluate critically the mathematical knowledge with which he has been equipped. He has the right to raise questions as to what values may be attached to the study of mathematics and what place, if any, the subject should have in any educational program.

[9] J. L. Mursell, *op. cit.*, p. 806.

It is true that, out of a rich background of information and with an enthusiasm of inspired interest in his subject, the teacher of mathematics must be prepared to enlighten the public on applied uses and to caution against the danger of undervaluation if interest is concentrated *only* on uses that have been made and are being made to the oversight of uses which might eventually be made. This responsibility, however, does not lie wholly in the hands of the teacher. The National Council of Teachers of Mathematics, the Mathematical Association of America, and the American Mathematical Society have very definite responsibilities here also. The Council must continue its interest in the better teaching of mathematics in the elementary and secondary schools of our country. The Association should increase its interest in the better teaching of mathematics on the college and university level. More thought must be given to curriculum and teaching problems at this level, and the Association is well organized to accept and promote interest in this responsibility. The Society should, of course, continue its excellent promotion of mathematical research, but let its members not forget that the future research workers in pure and applied mathematics must find the fundamentals of their highly specialized abilities in our elementary and secondary schools. Let us beware lest the foundations of our dream castles of mathematical superiority be laid in the sands of superficial teaching and incidental learning. All three groups have three joint responsibilities which are of major importance to mathematics and the mathematical program in our schools. *First*, they must provide a teacher training program which will place in our schools teachers who are interested and sufficiently versed in mathematics that they are no longer restrained to tread only the travel-worn paths of tradition. These teachers should have had the pleasure of exploring many of the interesting by-paths of mathematical applications, and should be so experienced in mathematical lore that they even dare explore new trails for the fun and thrill of finding what is there. *Second*, the three groups must devote intelligent co-operative effort to the construction of a mathematical curriculum for the schools which is freed from the "sterility of the cultural materials which specialists hemmed in by their specialities have selected for boys and girls to learn."[10] This curriculum should by no means be devoid of such cultural materials, but the challenge of modern progress must replace much of the homage to moss-covered tradition as a criterion for the selection of curriculum content in our elementary and secondary schools. *Third*, the three mathematical groups should devote thought, time, money, and energy to the promotion of a program for acquainting the general public with the real values of mathematics. If these groups are not interested in having the man on the street appreciate mathematics as a principal contributor to the welfare of man and the civilization of the human race, then who should be?

We may stand on the Prime Meridian of mathematical development and look backward through the passing years of achievement and be moved to poetic eulogy over the sunset of accomplished wonders. Rather, let us look forward to the challenge of the dawn of future progress. As we look to the immediate future through the dark clouds of world stress and sorrow, we need to make plans for the brighter days to come. What should we be doing that we may continue to march forward? Teachers of mathematics and the three major mathematical organizations have been making constructive steps in the direction of improved mathematical instruction in our schools. The present emergency has given us a helping hand. What is the next step forward? To find the inspired answer to that question we should all be directing our best efforts. Every individual or organization is worth just so much as the things are worth to

[10] J. L. Mursell, *op. cit.*, p. 806.

which their most earnest efforts are devoted. May we as teachers of mathematics and members of these respected organizations of those interested in mathematics not be modern Neros and fiddle while the world burns. Let us be concerned to discharge whatever patriotic duty it may be our privilege to perform, and let us also project our professional thinking beyond these dark days of war and dis order to the formulation of that mathe matical curriculum which might functio most effectively in the educational pro gram of the future designed to construc an ordered civilization out of the presen débris of conflicting emotions, confused interpretations, and grievous wrongs.

Regional Meeting Program of the National Council of Teachers of Mathematics at Cincinnati

Sponsored by the Mathematics Club of Greater Cincinnati. Masonic Temple, Feb. 13, 1943

9:30 Registration and Assignment to Luncheon Tables

10:00 General Meeting. J. V. Naugle presiding
The Place of Mathematics in Education
Walter L. Collins, Director
Department of Instruction
State Department of Education
Columbus, Ohio
What Mathematics is Needed in the Emergency?
Captain G. L. Dosland
U. S. Naval Training School
Miami University, Oxford, Ohio
Revisions in Current Subjects Made Urgent by the Present Emergency.
H. C. Christofferson, Director
School of Education
Miami University, Oxford, Ohio

2:00 Senior High School Section
Swet Nankovitch presiding
What Mathematics Teachers Can Do for our Future Aviators.
Bradley Jones Pro. of Aeronautics
College of Engineering
University of Cincinnati
Vectors and Their Uses.
Louis Brand, Head of Dept. of Math.
College of Engineering
University of Cincinnati
Discussion, under leadership of H. C. Christofferson

2:00 Elementary and Junior High School Section
Marie Hunter presiding
What May a Sixth Grade Graduate be Expected to Know about Mathematics and What Are Main Features of the Junior High School Program?
R. L. Morton, College of Education
Ohio University, Athens, Ohio
How Can We Improve the Teaching of Mathematics in the Upper Elementary Grades?
Fannie J. Ragland, Director of Upper Elementary Grades, Cincinnati Public Schools
Discussion, under leadership of J. V. Naugle.

12:15 Luncheon. Each table will have a discussion topic and leader as follows:
1. Vectors in High School Algebra and Geometry. Louis Brand.
2. Vitalization of Geometry through Aeronautics. H. C. Christofferson.
3. The Approach to Mathematics in the Elementary School. Walter L. Collins.
4. Mathematical Background for Technical Work in the Navy. G. L. Dosland.
5. Problems in Flying. Bradley Jones.
6. The Effects of the War on the Mathematics Program. R. L. Morton.
7. Understanding Numbers. Fannie J. Ragland.
8. Arithmetic in War Plants. Fannie Barnett.
9. Visual Aids in Mathematics. Lauretta S. Davidson.
10. Teaching Meteorology. Clarice Hobensach.
11. Teaching Navigation. Mildred Keiffer and Gladys Kloak.
12. Why has the Present Emergency Found Our Mathematics Teaching Inadequate? J. V. Naugle.
13. Should We Set Up Minimum Requirements in Arithmetic for High School Juniors and Seniors? Bertha Oechsle.
14. Making Numbers Meaningful in the Junior High School. Carmille H. Rush.
15. What Are The Important Outcomes of High School Mathematics? Dorothy Webster, John Haag } college students

Reservations for the luncheon including your first and second choices for a table should be sent to Miss Marie Hunter, 316 Joselin Ave., Cincinnati, not later than Tues., Feb. 9. Luncheon will cost $1.00.

The Double Track Plan of High School Mathematics

By HARL R. DOUGLASS
College of Education, University of Colorado, Boulder, Colorado

THE PREVAILING high school curriculum in mathematics was formulated very much much as it now exists in the quarter century immediately following the Civil War —1865–1890. In 1890 there were about 250,000 boys and girls in high school—about one in ten of all youth of high school age. Less than three per cent of young people were graduating from high school between 1880 and 1890. High schools were almost always looked upon as "prep" schools for "getting" the "bright boys" ready for college. The present high school curriculum in high school mathematics was built for those few who went on—the alber pupils, the college preparatory pupil, the future engineer, physicist, and teacher.

Since 1890, the percentage of youth of high school age attending high school has increased from 10% to 70%. In high school today we have 7,000,000 boys and girls, less than 18% of whom will go to college and less than 10% of whom will stay in college more than two years. The others will go into all walks of life as clerks, truck drivers, waiters, farmers, factory, mine, and mill employes, or will be the wives of these. At least a third of these do not have the ability and probably another twenty per cent can not be sufficiently interested to learn enough algebra, geometry, or trigonometry to be able to make use of it.

Yes this is truly a mathematical world for everyone. Everything we do seems related, in some way, to measurement. For a thousand years the world has been growing more and more mathematical. For a thousand years measurement and computation has been playing an increasingly significant part in the lives of everyone. In the last century this tendency has been rapidly accelerating. The tendency to become at once the creatures and the masters of precise quantitative thinking and action is a rapidly accelerating phenomenon.

A variety of factors, social and otherwise, have contributed to this tendency. Developments in transportation have played their part in many ways—time tables, the mathematics of construction, repair, purchasing, and operating of automobiles. Specialization in agriculture and other vocations have increased enormously the volume of business transactions—the selling of labor, farms, and other products, and the purchasing of practically everything for consumption in the home. And finally, the present crisis has revealed the importance, in total war, of quantitative literacy in the citizen.

The self-sufficient home has disappeared and in its place has come a producing and a purchasing unit which is compelling us to think in precise, quantitative terms and manners. The housewife of 1880 might well think of food in approximate terms of a hundred or so jars of fruit, seven or eight hams, fifteen or twenty hens, and a cow. She was under no necessity of calculating what was the best buy. Only for a small portion of her purchases was she compelled to make decisions based upon prices tacked on shelves for items of food of various sizes. The prices, as a matter of fact, were not tacked on the shelves or otherwise made known to her unless she asked. Cooking and recipes were vaguely or less exact in so many eggs, so many cupfuls, so many tablespoonfuls and the like, but weighing had not yet arrived and scales were rarely found in the kitchen of 1880.

In like manner other developments in

communication, in the knowledge of diet and other phases of health, in commercialized amusement and recreation, in artistic and aesthetic fields, in installment buying have actually increased the need for present quantitative thinking.

In the realm of social security; in fire, life, hospital, medical, and automobile insurance; in annuities and other forms of providing for the future; inn ew forms of taxes and the extension of old forms that make democracy indeed a cooperative form of spending, our need for computational thinking has increased many fold.

Problems of travel did not require the quantitative treatment that they do today. The dashboard boasted no array of mathematical gauges to measure this, that, and the other, as does the instrument panel of automobiles and airplanes. There was no calculation of how many miles to the bushel of corn or to the bale of hay. In the construction, repair, and upkeep of buggies and wagons, measurement in thousandths of an inch was not dreamed of.

In an interesting paradox to this very marked increase in the mathematical needs of all of us, there has been a constantly decreasing percentage of children in school above the eighth grade studying mathematics. It is worth noting that in 1910 more than three-fourths of all high school students were enrolled for a class in mathematics while in 1940 the proportion of high school pupils enrolled for a class in mathematics is less than one in three. WHAT A COCK-EYED WORLD IT IS IN WHICH, AS THE NEEDS FOR MATHEMATICS BECOMES GREATER AND MORE UNIVERSAL, THE SMALLER IS THE PERCENTAGE OF HIGH SCHOOL STUDENTS STUDYING IT. *Mathematics as we have* taught it in the high school has been confined largely, in its practical values, to the vocational needs of a small group of college trained workers—chiefly engineers.

The mathematics taught in the elementary school has indeed been of the type needed, in practice it has proved inadequate. The children there are too young, too immature. They have too little experience to understand or to be interested in the life problems calling for mathematics. Too little time has been available to result in mastery. The applications are too complicated for twelve year olds. We see this when seventh graders attempt to calculate the comparative returns from various kinds of investments, or whether it is cheaper to buy a house at so much in a city where taxes are on the average so much—with repairs and depreciation likely to be so much, etc.—or to rent one of similar size, etc., at so much a month.

It seems quite illogical to attempt to meet the mathematical needs in the elementary school, and then to confine instruction in mathematics above the elementary school to a type of mathematics which is designed for a special group going on into further study of mathematics or science, or into engineering in college. At a time when the great majority of students did not go beyond the elementary school, it was probably defensible to crowd into the grades the mathematics needed in everyday life. BUT WITH THE GREAT INCREASE IN THE NUMBER GOING ON TO HIGH SCHOOL, THE NECESSITY FOR THE PREMATURE INTRODUCTION OF DIFFICULT ARITHMETICAL CALCULATIONS DISAPPEARED AND APPROPRIATE ADJUSTMENTS ARE CALLED FOR.

Today when only one high school freshman in ten will finish a year in college, and one high school senior in five will enter college, the failure to offer in high school mathematics other than algebra, geometry, and trigonometry is not defensible from the point of view of the student nor from that of the best interests of mathematics. Even the college-going student needs a much greater mastery of arithmetical processes—particularly ratio and proportion, and computational operations with common and decimal fractions—as

practically every professor of chemistry, physics, accounting, and engineering will testify.

The arbitrary location of all arithmetic in the first eight grades, and only algebra, geometry, and trigonometry in the last four grades conforms neither to the educational needs of the present age nor to what we know of psychology—of the abilities and interests of young people of the ages concerned.

Many high school teachers of mathematics would, it is true, find it necessary to review and extend their mastery of arithmetic and to acquire much more knowledge concerning the applications of arithmetic to various phases of life. Yet this does not seem an unfair burden to ask teachers to assume in the interest of the pupils and the nation they purport to serve.

Perhaps as influential as any other force is the questionable attitude taken by many teachers of mathematics that it would be beneath their dignity as majors in mathematics—students of calculus and differential equations, to teach mathematics other than algebra, geometry, and trigonometry in high school. I can well appreciate this position, its naturalness and its dangers. When I graduated from the University, having majored in mathematics and with the innocence and ignorance of life outside the classroom typical of young graduates, I, too, wished to demonstrate my knowledge of "higher" mathematics and my ability to teach it.

There should, in addition to the algebra-geometry-trigonometry sequence be developed in the high school another series of offerings in mathematics for the great majority of pupils. This should be composed largely of arithmetic and its applications to all phases of life, home, shop, farm, business, health, travel and transportation, social security, etc. It should also include considerable geometry of construction and measurement of surfaces and volumes. It should also include correlated in its applications with arithmetic and geometry, at least six or seven months of algebra, literal and negative numbers, much work with simple equations and formulae, a large variety of applications particularly to science, considerable work with graphical representation, and some work with simpler statistical constants and procedures.

This sequence for the general school population might well be organized in a variety of ways. Among them might be mentioned the following:

Plan One

For the abler pupils

1. Enriched arithmetic emphasizing decimal fractions and percentage in grade seven.
2. One semester each year in grades eight through twelve of general, practical mathematics, largely arithmetic, including about ⅓ of first year algebra, some geometry, and a little numerical trigonometry.
3. For *abler pupils* who are interested:
   a. In smaller schools, i.e., less than 200 or 250 pupils; 1 year of algebra in the eleventh grade—3rd semester algebra, and 1 year of plane and solid geometry in the 12th grade.
   b. In the larger schools, the same sequence but beginning in the tenth grade and including also a semester each of algebra and trigonometry.

For the others

Plan Two

For the abler pupils

1. Three years of general mathematics in grades 7, 8, and 9, largely arithmetic, *required* of all, and including approximately one semester of algebra.
2. The same sequence as in plan one for the abler pupils, except that the first year of algebra would include what is now taught as second and third semester algebra.

For the others

3. Several elective one semester courses in grades 11 and 12 in applied mathematics—largely arithmetic but including applications of simple algebra and geometry.

Plan Three

For the abler pupils

1. General mathematics, very largely arithmetic in grades 7 and 8.
2. For the abler pupils, one of the sequences indicated under 3 in Plan One.

For the others

3. Tenth or eleventh grade—a two-semester course in general mathematics involving about a semester of algebra and the more complicated applications of arithmetic, required of all not taking the course for the abler pupils.
4. Grade twelve—two elective semester courses—largely applied arithmetic.

Already more progressive high schools by the score have moved in these directions as indicated by the report of the California State Department of Education in its recent bulletin, "Some Practices in California Secondary Schools." All mathematics teachers who are really interested in meeting the mathematical needs of the boys and girls in our schools and in serving the interests of mathematics in the schools should examine this publication. Those teachers or administrators who prefer to keep in the ruts for fear they may not be able to find the way back, I advise to shun the publication. It is dangerously inspiring.

I have in recent weeks been studying the recommendations of the Navy concerning the increased need for mathematics. The following conclusions seem to me to be sound.

1. There is obviously need for more instruction in mathematics both of military and for allied civilian purposes.
2. The Navy recommendation in their bulletin on that subject suggests that 40% of the courses in high school be *arithmetic*.
3. We must not neglect civilian needs, as few if any of our freshmen and sophomores next year will participate in the war or in allied engineering jobs. The civilian needs are permanent and will be present when the war emergency has passed.
4. There is great need, temporary just now, for courses in algebra, arithmetic, and trigonometry for seniors, graduates, and junior college students.

It is not worthy either of teachers or of mathematicians to resort to insupportable demogogic appeals or sales pressures in an effort to exploit feverish and superficial thinking during the war for the purpose of increasing enrollments in mathematics in conventional courses. Not only does this approach cheapen us in our own self-esteem and mutual regard but it serves to create in the minds of school administrators and other educators a lack of confidence in high school mathematics and

teachers of mathematics and a conviction that the study of mathematics does not seem to produce logical and scientific thinking. What is really needed is not so much sales pressure but more thought and effort in revising our courses to meet real and increased needs.

The present emergency, throwing, as it does, the spotlight on the importance of mathematics in all forms of cooperative or technical enterprise, offers us an opportunity not only to recover lost ground, but to establish the importance of our field in a realistic way that cannot be denied, thus relieving us of the constant necessity of apologizing for our failure to educate for the mathematical needs of the great mass of American men and women.

Reprints Still Available

Tree of Knowledge	5c
The Science Venerable	5c
Grade Placement Chart in Mathematics for Grades 7 to 12 inclusive	10c
The Ideal Preparation of a Teacher of Secondary Mathematics from the Point of View of an Educationist. Wm. C. Bagley	10c
Value and Logic in Elementary Mathematics. Fletcher Durell	10c
Modern Curriculum Problems in the Teaching of Mathematics in Secondary Schools. W. D. Reeve	10c
Proposed Syllabus in Plane and Solid Geometry. George W. Evans	10c
A Plan for Meetings of Mathematics Teachers in a High School. H. P. McLaughlin	10c
Report of the Committee on Geometry	10c
A Study of Procedures Used in the Determination of Objectives in the Teaching of Mathematics. J. S. Georges	10c
Report on the Training of Teachers of Mathematics. E. J. Moulton	10c
Professors of Education and Their Academic Colleagues. W. C. Bagley	10c
Crises in Economics, Education, and Mathematics. E. R. Hendrick	10c
Arithmetic and Emotional Difficulties in Some University Students. C. F. Rogers	10c
The National Council Committee on Arithmetic. R. L. Morton	10c
Lots of 12 or more	5c
Mathematics and the Integrated Program in Secondary Schools. W. D. Reeve	15c
Informational Mathematics Versus Computational Mathematics. Charles H. Judd	15c
The Cultural Value of Mathematics. Aubrey J. Kempner	15c
Whither Algebra. William Betz	15c
The Necessary Redirection of Mathematics, Including Its Relation to National Defense. William Betz	15c
A Proposal for Mathematics Education in the Secondary Schools of the United States. W. D. Reeve	15c
The Present Situation in Secondary Mathematics, with Particular Reference to the New National Reports on the Place of Mathematics in Education. William Betz	25c
Traders and Trappers. Carl Denbow	25c
The Algebra of Omar Khayyam. Daoud S. Kasir	50c
Three Major Difficulties in the Learning of Demonstrative Geometry. R. R. Smith	$1.00

The above reprints will be sent postpaid at the prices named. Address

THE MATHEMATICS TEACHER

525 W. 120th Street, New York, N.Y.

The Objectives of Tenth Year Mathematics*

By HARRY EISNER
Manual Training High School, New York, New York

THE NEW course in seventh, eighth and ninth year mathematics has now reached the 8B grade in all elementary and junior high schools. The graduates of the elementary schools will be in our high schools next September. The great majority of these pupils will start their high school mathematics in February, 1944. Will we be ready for them?

Already, teachers and supervisors of mathematics in New York City are thinking and planning ahead. Such plans as are being formulated are necessarily general and tenuous. One fact is emerging, however, with increasing clarity and emphasis: the new tenth year will not be the familiar pabulum in demonstrative plane geometry. Substantial, perhaps radical changes are being dictated and accelerated by the force of events and by a growing appreciation of the fuller and richer possibilities of our subject for our boys and girls.

The present junior high school syllabus is the first practical and large scale implementation of a new philosophy of the teaching of secondary school mathematics which had its inception in the National Committee Report in 1923. Until now, we can say frankly that we have been paying little more than lip service to its creed. We have, to a large extent, preserved the ancient framework of our compartmentalized mathematical offering: a year of algebra, following this, a year of geometry, then a continuation of algebra, then trigonometry, a further continuation of algebra, and finally more geometry. It is true that we have reduced the number of required types of factoring and simplified the form of the denominators of the fractions that we require our pupils to add. It is true that we have introduced a little functional thinking, as typified by the question "What happens to A in the formula $A = \frac{1}{2}bh$ if b and h are doubled?" It is true that we have reduced the number of propositions in geometry which we demand that our pupils reproduce. But has the fundamental spirit of the content of our curriculum changed radically since 1915? Select at random an examination paper we give to our pupils. What do we see? More likely than not, the same manipulative techniques as of yore crowd the algebra paper, the same verbal problems falling neatly in time honored grooves—an interest problem, a work problem, a coin problem—, the same reproduction of stock propositions, the same abstract numerical applications, the same formal constructions. If these are the things we ask our pupils to do for us on examinations, then these are precisely the things we stress in our teaching. Look at an examination paper and you have a mirror of classroom activity.

It would be unfair, however, to assume that individual teachers and individual schools are not breaking away from this formalism. Thoughtful elements in our ranks are pointing ahead in the direction of enlightenment and progress by experimenting with new materials and new methods. There are signs, too, that the Regents' examinations are beginning to reflect the new tendency, particularly in connection with the last group in the plane geometry paper.

We must not be satisfied, however, with just a stab here and there, or a limited readjustment of the course of study in a few schools. The problem is larger and broader in its spirit and application. What has now been accomplished for the ninth year is only the forerunner of what I hope will be a far-reaching reorganization

* A paper presented at the Joint Luncheon of the Mathematics Chairmen's Association and the Association of Teachers of Mathematics in New York City at the Hotel Astor, March 21, 1942.

of the three succeeding years. We must not fail to bring our upper courses in line with a sound, progressive philosophy. It may not be an exaggeration to state that the very future of mathematical instruction is at stake and will be determined by the efficacy of our efforts. The immediate problem is Tenth Year Mathematics.

How shall we as teachers undertake this tremendous project? Two antipodal procedures suggest themselves: one is lazy and superficial, the other is thoroughgoing and fundamental. The former may be termed the scissors and paste method; the latter, the functional process. Here we have our choice between a patching up of our present syllabus in plane geometry, that is, filling a gap here and there, or eliminating this topic or that, each change being a grudging concession to new demands, or, on the other hand, a reconstruction of the entire tenth year based upon a sound interpretation of accepted objectives of mathematical instruction. Every student of curriculum construction deprecates the first alternative and extolls the second. I am sure that every thoughtful teacher of mathematics agrees with this view.

Webster's dictionary defines an objective as a point forming an object of attraction or end. Unless we can visualize the ends that we desire, that is, the outcomes to be achieved, how can we plan intelligently the materials and methods of instruction which are presumably the means for the realization of these ends? Does an engineer drive a train without a clear realization of its destination? How meticulously plans are laid by the railroad with regard to route, speed, equipment, repairs, and a host of other factors to achieve the fundamental objective, namely to receive and discharge passengers at the right place and at the right time! The education of our youth is a far more complex process than driving a train; the path of learning is beset with many more obstacles and uncertainties. The human element is less susceptible of prediction or determination than inanimate matter and energy. Well, that is all the more reason why we as educators must strain every effort to guide our perplexing charges in the general direction of worthwhile goals.

When a teacher understands and accepts a formulated objective of a course of study and sees it in relation to well defined units of instruction he has taken a long step toward incorporating the spirit of the objective in his teaching. Unfortunately, many of the published objecjectives of mathematics courses are stated in such general terms that their application to specific topics is difficult and obscure, or the objectives are expressed in such detailed fashion that they degenerate into mere items of subject matter to be mastered without regard to the larger ends or goals of instruction. To say, for example, that an objective of mathematics is to develop habits of clear thinking or good citizenship without mentioning specific applications to units of subject matter or methods of teaching is neither constructive nor illuminating to a teacher whose lesson for the day is the proof of the proposition that a tangent to a circle is the mean proportional between the whole secant and its external segment. To assert that an objective in the teaching of geometry is the ability to bisect an angle with straight edge and compasses without relating this skill to a larger purpose or need may well result in feeding a specific minor performance but starving its rich background of insight and application.

I have been asked to address this panel on the objectives of Tenth Year Mathematics. I should like to emphasize the words *Tenth Year Mathematics*. Let us not assume that we are planning a course in geometry. Geometry, as we ordinarily conceive this subject, may turn out to be the major constituent of the course of study of the tenth year, or it may not, depending upon the determining objectives. We must be careful not to place the cart before the horse. Another point I should like to make at this time is that the objec-

tives which I shall propose represent my personal point of view. I am not speaking officially for the Standing Committee which has not completed its careful consideration of this problem.

In the year 1923 the National Committee on Mathematical Requirements presented its notable report. In vision and outlook it was far ahead of the prevailing philosophy and practice of the day. As time went on, the influence of the report on courses of study slowly but surely increased. Here in New York City, the planning, preparation and adoption of the new course of study for the seventh, eighth and ninth years is a definite outgrowth of the work of the National Committee and of the intellectual awakening which the report engendered. Today, nearly two decades after the publication of the report we may still admire its classic formulation of the fundamental objectives of the teaching of mathematics. Here is its text: "The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and action which will make these powers effective in the life of the individual." The conclusion is inescapable that the primary thought in the minds of the National Committee was that mathematics should be taught, not as an end in itself, but as a means of developing in our pupils those appreciations insights and powers which would enable them to live a more useful and more effective life. Correspondingly, those concepts and skills which tradition or inertia has kept in our course of study but which are definitely non-contributory to fuller and more efficient living should not be retained at the expense of more vital material which might otherwise have been included.

The two statements I have just made constitute a very general expression of the objectives of the teaching of mathematics as I see them. In the balance of this paper I shall attempt to clarify and expound this point of view and indicate its application to the development of more detailed and concrete objectives for the tenth year.

Educational objectives may be classified for convenience into two types: the personal or psychological objectives, which refer to desired changes or outcomes in the individual, and the impersonal or curricular objectives which relate to ends expressed in terms of subject matter or class management or administrative procedure. The primary objectives are the personal or psychological ones. In order that these personal objectives may have greater significance to the listener, I shall indicate in each case the areas of subject matter and to some extent the methodology or administrative devices which seem to me to be causally related to the desired ends.

In the first place, it is obvious to any careful observer that the curricular trend in all academic subjects is definitely in the direction of a closer relationship and proximity to every-day living. It is becoming increasingly recognized that a subject should owe its place in the curriculum to something more than the development of insight and mastery directed at abstract elements bearing little or no relation to the world in which we live. The conviction has grown until it is now a near certainty that a subject has no business in the high school curriculum if it can not or will not make a definite and worthwhile contribution toward a better interpretation of life, and toward a better adjustment to life. Mathematics has not failed to accept this challenge in so far as the work of the ninth year is concerned. Now that the time has come to re-examine our tenth year course we should re-explore the possibilities of a closer tie-up with life than has been attempted or secured in the past. I submit therefore, as the first major objective of the tenth year course, the development of appreciation of the place of

geometry in life and the ability to use geometry in practical life problems and situations. What are the curricular implications of this objective? First of all, there should be a continuation, on a higher level than was attained in grades 7A–9B, of the observation and recognition of geometric two dimensional and three dimensional forms in nature, in industry and in art. Such recognition should include the understanding of the different types of symmetry and the observance of the existence of relations such as parallelism, congruence, similarity. The construction by the pupil of geometric figures, including solids, and the measurement and comparison of elements in these figures will strengthen the understanding of the basic relations and serve as the foundation for the discovery of more subtle relations by deductive means. In this connection, also, attention should be given to the evolution of our knowledge of common geometric figures and their properties from the time of Ahmes and his Papyrus to the present day.

The ability to use geometry in practical life problems and situations may be developed through practical applications, many of which are cited in text-books and treatises. Problems of this type fall into three subdivisions:

1. The appreciative problems,—those applications contributing to the development of appreciation of the value and power of geometry, although not of common, every-day occurrence and not vocational in an exclusive sense. As instances of such problems may be mentioned the division of a sheet of paper into any number of equal parts by use of a supplementary strip of ruled paper, the use of the pantograph to magnify or minify a map, and simple geometric problems related to avigation and navigation.

2. The Functional applications,—those which arise in every-day life situations. Illustrations of this type are the understanding of the use of triangles to establish rigidity of common objects or structures, the determination of the areas of floor spaces, the detection of fallacies of improperly constructed pictograms, such as occasionally appear in newspapers and magazines.

3. Vocational applications. These are of specialized interest and generally unsuitable for the academic pupil, except that to a limited degree they are useful for their exploratory vocational value. Examples of these applications are those relating to roof shingling, brickwork, board measure, painting, paper hanging and printing.

The most valuable practical problems are obviously those of the functional type. While, unfortunately, such problems are far from numerous, there are many practical problems of the appreciative type which will appeal to pupils and which have excellent motivating possibilities. Certainly the determination of the content of the tenth year course with respect to geometric concepts and relations should be influenced to a considerable degree by the educative value and the quantity of practical applications which may be associated with the proposed topics.

Important and interesting as the practical applications of geometry are, they would hardly justify, by themselves, the spending of a full year of the pupil's high school career. The applications should be understood in the light of principles from which they are evolved. The reasoning called for in these problems is only a part, and a small part, of the thinking experiences which should be offered to our students through the medium of mathematics. I propose, therefore, a second major objective for the tenth year: the development of the ability to think logically. No one will question the paramount importance of this objective. Many persons, including teachers of mathematics, are questioning the results that are crowning our weary efforts to realize this objective. Perhaps a few pointed queries will serve to indicate some weaknesses in our present tenth-year set-up.

Why is so much time devoted to direct deductive thinking and so little time given to other equally important forms of thinking, such as inductive reasoning, indirect reasoning and analysis? Why do we plunge into formal demonstration before our pupils understand the nature of proof? Why is it necessary to teach sequence in geometry by means of a large and unwieldy organization of over a hundred theorems when this concept might be much better developed through a selected series of short units such as the parallel line theorems in Book 1, the angle measurement theorems of Book 2, the propositions leading to the Pythagorean theorem in Book 3, and the area theorems of Book 4? Why do we attempt to teach our pupils at least twice as many geometric facts as they can assimilate? Why do we limit ourselves almost exclusively to geometric content in the development of logical thinking when there are so many excellent possibilities of illustration of correct and incorrect reasoning in every-day situations which are basically similar to the geometric instances? Are we tied down by the force of inertia which leads us inexorably to keep on doing what we have been doing for years? Have we been placing the acquisition of subject matter and skills higher than the mental development which the concepts and skills are presumably intended to effectuate? This last query strikes at the heart of our predicament, in my opinion. Putting the situation symbolically, we have been declaring that we must, at all costs, teach definition *A*, proposition *B*, formula *C*, and application *D*. Having securely rooted *A*, *B*, *C* and *D* in the course of study, we scout about for a pedagogical justification for their inclusion. Sure enough, we soon find an elastic, general objective *X* into which *A*, *B*, *C* and *D* neatly fit. Hence *A*, *B*, *C* and *D* remain firmly entrenched in the course of study. On the other hand, a sound theory of curriculum construction demands the converse procedure: first the acceptance of *X* in the light of broad, fundamental criteria, then the search for and the inclusion of the material that will best achieve *X*.

Time does not permit a detailed analysis of the curricular implications of this second major objective A brief statement of one much neglected aspect, reasoning in every-day situations, must be made before passing on. It is the claim of some educational leaders that the development of reasoning ability in common life situations is not the concern of teachers of mathematics. Reasoning in life, they say, is essentially different from reasoning in geometry, and far more complex. With this point of view I find myself in essential disagreement. What better illustrations of common errors in every-day dialectics: arguing in a circle, faulty generalizing from insufficient data, using ill-defined terms, assuming the truth of the converse, non-sequitur reasoning, failing to consider all the alternatives in indirect reasoning, omitting authoritative evidence for the support of an assertion, ignoring or contradicting given data,—what better illustrations of these may one desire than may be found in geometry? All that is needed is to prepare a sizable number of simple errors in every-day reasoning of the types mentioned above and present them in association with their geometric counterparts.

The third and last major objective is the development of certain desirable attitudes and general traits. In this category would be included respect for knowledge, good workmanship, precision and accuracy, open-mindedness, social-mindedness, love of discovery of new facts and relationships, courage for truth, the will to perfection and reverence. An excellent discussion of how mathematics may serve as a medium for the development of some of these traits may be found in a report on objectives prepared by the Standing Committee on Mathematics in 1934. It must not be assumed that these qualities, which have been termed the "intangibles" are the exclusive domain of mathematics.

Any teacher of the social studies or English or science will tell you at once that these are precisely the qualities of mind and spirit which emanate from his subject. It is not necessary to indulge in argument as to which subject can make the greater contribution in this area. This is a time for co-operative effort and harmony, not factionalism and strife. It is my conviction, however, that desirable personal attitudes such as those enumerated have their basic origin in the quality and the enthusiasm of the teaching, in the adoption of appropriate classroom procedures and activities, and in administrative adjustments whereby levels of ability of the student body are recognized and provided for. Even the Queen of the Sciences can be presented in such a manner as to arouse the negative attitudes of inattentiveness, indifference, slothful performance, dishonesty and antagonism. And the lowliest subject, figuratively speaking, can glorify and realize those essential spiritual qualities that we cherish.

In conclusion, may I re-emphasize that a course of study for the tenth year constructed on the basis of the objectives proposed in this paper will differ significantly from the familiar course in demonstrative geometry. The number of required propositions and original exercises of the common deductive type would be considerably decreased. Time would thus be saved for increased work in indirect reasoning, in applications of the inductive method, and in errors of everyday reasoning. Spatial insight would be extended to three dimensions. Practical applications in the physical world would constitute a major goal rather than a by-product which busy teachers may omit with impunity. A closer correlation of algebra and geometry would be accomplished as a result of the expansion of the method of analysis. "But," it will be objected, "this is not a course in plane geometry." True, but neither is the new ninth year course a course in elementary algebra. The crucial question is "What is the best material which we can offer to our tenth year pupils who have just completed the new ninth year course and who may never take any additional mathematics beyond the tenth year?" This is the vital problem to which the best thought of teachers and supervisors of mathematics should be directed during the coming months and years.

The Seventeenth Yearbook!

A Testimonial

TEACHERS who are looking for vital applications of mathematics to the war effort will find the *Seventeenth Yearbook* of the National Council of Teachers of Mathematics one of the most helpful of recent publications for their purpose. It seems that those responsible for this book must have had some premonition of the demands the war would make upon mathematics, for the book contains a wealth of applications of mathematics to mechanics, aeronautics, navigation, and other technical fields, besides the usual applications to life situations. And the applications are arranged in such a way that the reader can find what he is looking for with a minimum expenditure of time and effort. No teacher of mathematics desirous of making his best contribution to the war effort can afford to be without a personal copy of this book.—BENJAMIN BRAVERMAN, *Chairman Department of Mathematics, High School of Commerce, New York City.*

Statistics—The Mathematics for Social Problems*

By Douglas E. Scates
Duke University, Durham, North Carolina

Quantitative considerations, in ways more fundamental than we are prepared immediately to realize, are playing an increasingly significant role in our everyday lives. In the mechanical world, which we have to some extent reordered according to our convenience, our dependence on mathematics is clear and unmistakable. In the social world, which we have not yet brought into wholly creditable order, we have only begun to recognize that facts must be faced in quantitative terms. World conditions, however, are rapidly forcing upon us an era in which planning, both for individual affairs and for society at large, is heavily weighted with quantities.

We are now experiencing in new ways the significance of quantitative facts in such areas as national expenditures, taxes, "manpower," travel, agricultural output, industrial production, consumption by the army, consumption by millions of underfed persons across the ocean—and so on. Statistical facts and concepts are currently in the main headlines of newspapers; and we have lately come to expect index numbers, trend curves, bar charts, and other somewhat technical representations of change, as a part of our daily reading. National and local deliberative bodies must procure extensive data to guide them, and we are gradually realizing that quantitative facts are essential to all policy making—in government, industry, education, organizations, and even in home and personal planning.

With this increased emphasis upon quantitative thinking in all aspects of social living, we must undertake a corresponding development in our high-school curriculum. If we are to have citizens who are capable of carrying on the traditions of democracy under the new rigors which are permeating our society, we must begin at once to prepare young citizens for their demanding task. Effective training of young people will consist of participation in high school (and college) in precisely the same sort of thinking which, as adults, they will be called upon to do, namely, engaging in statistical attacks on social problems.

The customary mathematics of high school (and college) was designed to serve the needs of the physical sciences; it serves these needs well. Social problems, however, particularly the non-vocational ones, have largely been neglected in mathematics. Indeed, the high-school student gets almost no stimulus from his mathematics for attacking the most important problems that his society regularly has to face. The reason is that social data call for a different form of attack; and the emerging prevalence of quantitative conceptions in personal and social areas has created the need for a new form of mathematical training. The kind of mathematics which has been developed for dealing especially with these areas of rapidly growing need is statistics.[1] There are opportunities latent in the statistical method for accomplishing in the social

* Paper read before the Second Institute for Teachers of Elementary and Secondary Mathematics, Duke University, Durham, North Carolina, June 16–26, 1942.

[1] Statistics is the branch of mathematics that is concerned primarily with distributions—the breaking down of an aggregate of units according to categorical or quantitative classes. Statistics deals largely with the representation and characterization of distributions, and with their dependence on observed factors or on factors associated with time, space, and chance.

areas what traditional mathematics[2] has made possible in the physical areas. The pressing need is to develop an appreciation and an understanding of the statistical approach on the part of a large portion of our society.

Statistics is truly a socialized and a socializing form of mathematical thinking. It requires that one consider not only himself but his class; and not only his own class, but all classes; for any individual becomes, in statistics, but one of a class, and the class becomes but one segment of a distribution. The statistical form of thinking inevitably draws attention from the individual to the group—either because of direct interest in the group or because the significance of any value can be revealed only through relating it to the group. Statistical thinking then goes further and considers the observed group as but one of an indefinitely large number of similar groups produced by the shifting combinations in an underlying set of causes. As in astronomy one thinks from the individual star to the cluster, then the galaxy, the supergalaxy, and finally, external space, so in statistics one views the individual case in the increasing perspective of ever larger groups, running, if one pleases, throughout indefinite time.

Objectives for Statistics Teaching

Since high schools furnish the terminal education for so many of our young people it becomes important that they lay a groundwork of appreciation and understanding for the quantitative approach to the solution of social problems. We must not think primarily of producing vocational specialists; in the case of the more intellectual pursuits that can scarcely be done on the high-school level. We can of course lay a background for vocational work while we are attempting the far larger and more universally important task of training citizens. Our selection of objectives for a high school course in statistics will be guided accordingly. Three objectives seem to stand out prominently:

1. To produce statistical literacy. That is, the ability to read diagrams and maps; a "consumer" understanding of common statistical terms, as average, per cent, dispersion, correlation, and index number. Such ability is needed by newspaper readers. Many young people acquire a large degree of statistical literacy from regular mathematics and other subjects; but some students who are otherwise intelligent carry with them throughout life a marked deficiency in this area. This objective is an elementary one and should be espoused for all pupils.

2. To accustom young people to doing their thinking about personal and social problems in terms of quantitative facts wherever appropriate. This means that they would come to expect facts, expressed quantitatively when possible; that they would seek such facts when not already in hand; that they would hesitate to come to conclusions when facts are not adduced; and that they would learn to look questioningly at facts as to their validity and adequacy. Such habits constitute the primary need of our people for statistics. This objective should be attempted for all who can be led to think with some care.

3. To familiarize young people with the processes of gathering data, and the elementary modes of interpreting them. The great majority of people grow up with the idea that handling figures is somebody else's business; but the citizen who is going to be active in his community needs often to gather data. He needs to break through the feeling of mystery concerning statistics and their sources. Even if he confines himself to available data, he needs to understand the methods back of the data he is using so that he can better evaluate them. If he is to feel at home in employing data in his thinking, he should

[2] The writer has no intention of implying a clear-cut dichotomy, but rather a marked difference in emphasis and appropriateness. The methods and areas overlap. Statistics is important in natural science; and general mathematics is important in many aspects of statistics and in financial problems.

more closely identify himself with the entire statistical process. This objective holds primarily for those who will be active in helping to guide the thinking of others.

For students of more than average ability, and who have an interest in the manipulation of figures, we should probably add a fourth objective, namely, some technical skill in calculating, and in drawing diagrams that meet somewhat exacting requirements of neatness, clarity, and accuracy. But we should bear in mind that the ability to produce refined results will be needed by relatively few, whereas a "consumer" understanding of data and the inclination to couch one's thinking in statistical terms are needed by everyone who graduates from high school.

The three major objectives emphasize outcomes which are on the order of orientations, appreciations, expectancies, dispositions to use, and feelings of familiarity. These relatively intangible aspects of learning are the main factors in determining what habits will persist and eventually find expression in action. In the training of citizens these are the kinds of outcomes that are of greatest importance. If we believe that we can have a better society through a greater degree of quantitative thinking, our goal becomes that of getting statistics into the everyday thinking of people. Such a goal requires emphasis on the dynamic aspects of learning.

For those who prefer to think of mathematics solely in terms of skills, and who do not care to give large consideration to the non-skill factors entering into social dynamics, I would urge the same conclusion, but would advance somewhat different reasons. If one would teach a skill to a large number of persons, one must have social support for the teaching or it will not continue. I would attempt to assure a market for my skilled persons before making any great investments in production. If we can cultivate in a generation of young people the demand for quantitative social facts then the need for persons with skill in producing and analyzing those facts is almost certain to be felt and met quickly. We cannot, however, say conversely that the training of many embryonic statisticians in technical abilities will promote a demand for their services. In any well-rounded concept of social progress, more value attaches to that teaching which will develop a consciousness of need and a determination to have that need met than to teaching which produces skills apart from a broad background of understanding and a social demand for them.

Statistics As a Normal Part of Problem Solving

If the objectives mentioned are accepted and held clearly in mind, it is probable that they can be realized by a variety of different teaching procedures. I should prefer to leave teaching methods to the individual teacher, who would choose methods which would attain the desired ends and at the same time fit in with the practices and philosophy of the school.

I would however point out that the greatest contribution to thinking about social problems will be made by that teaching of statistics which emphasizes its value in solving problems. This is an essentially different emphasis from drilling on calculational procedures or cultivating theoretical understandings. It is all too easy to learn the mechanics of calculation without learning how to interpret the results. The important thing in teaching any mathematics for practical use is to *teach that mathematics as a part of a stream of thought in which one engages when facing a natural problem.* If we fail to make mathematics *an habitual element in common thinking,* and present it only as a special technique with currents of reasoning and rules of its own, we leave the student greatly handicapped in his use of mathematics—except for doing research in the field of mathematics. Statistics is a basically practical study, and it should be

taught as a form of thought in solving problems if it is not to miss its opportunity.

Statistical concepts which are to function in everyday life must be stimulated by the same stimuli that start and guide one's problem-solving trains of thought. Statistical concepts must, accordingly, be taught in that fashion. They must be instilled as a framework of thought which springs to mind when practical problems are faced. Teaching must therefore begin with practical problems, and introduce the statistical modes of thinking as normal sequences in solving those problems. This must not be done once, but over and over —not primarily to drill on the technical skill but to drill on its appropriateness and naturalness as a dependable phase of solving problems. Such relations or sequences need to be developed in various fields of application, until the statistical concepts have been both habituated and generalized—generalized through their appearance and utility in a variety of different areas of content. We can then trust the student to carry on the process of generalizing until the skills have the widest possible utility for him. But we cannot trust this generalizing process to go forward so well if we teach the skill first as a perfectly general, relatively abstract tool, and then attempt to induce a variety of applications. Abstraction is psychologically the end product of growth and familiarity; it is not a suitable beginning point.

It is recognized that under any system of teaching both specific details and large patterns of thought must be given definite attention; neither area can be taken for granted if outcomes are to have the greatest utility. Statistics instruction in high school can either be made a separate course, or be made a phase of projects in other courses, so long as two conditions are met: (1) that the statistical concepts and procedures grow out of, and be well integrated with, content which is live and immediately significant to the student; and (2) that attention be directed toward the statistical aspects so as to make the student conscious of them, and so as to provide a reasonable amount of meaningful habituation. Neither the social studies content nor the mathematical approach should be so prominent as to obscure the other.

Sources of Statistical Material

Sources of material suitable for teaching statistics are, fortunately, abundant. I shall call attention to a few of them by way of illustration. We may start with population, as the most general social fact.

Population.—Census figures for one's own city, county, state, or the nation, can be made of great interest, and they hold almost unlimited opportunities for statistical representation and treatment. At the present time "manpower"—including womanpower—occupies a large place in our thinking. How large a fighting force can we support—and at the same time supply munitions and food to our allies, and maintain at home a minimum standard of wholesome living? Materials of considerable instructional interest may be produced by applying general (national) statements which appear in the newspaper to factual data on population and occupational distribution in one's own community.

If one lives or teaches in a city, the growth of cities during the past hundred and fifty years is a matter of real interest —and a fact of considerable social significance. The National Resources Committee[3] points out: "There has been a distinct shift in the Nation's status from a predominantly rural to an urban people, a development so swift as to be without precedent in the history of the world." (p. vii) A hundred and fifty years ago there were "a mere half dozen cities" and only 3 per cent of the total population

[3] National Resources Committee. Report of the Urbanism Committee, C. A. Dykstra, Chairman. *Our Cities: Their Role in the National Economy.* Washington, D. C.: Government Printing Office, 1937. 88 p.

lived in them. In 1930, for the first time in our history, more than half of the people lived in cities. "The family has grown smaller and the older-age group larger. American cities instead of maintaining a birth rate dufficient to reproduce themselves must recruit from the country. The conditions of rural life today are therefore the preconditions of urban living tomorrow" (p. vii). These are not merely statistical facts; they are social facts of great moment, and statistics is simply the tool for revealing them. Such basic facts are ready for further statistical elaboration and interpretation—and also for further social elucidation and ramification. One can go on, as the Urbanism Committee does, to note, in quantitative terms, some of the problems which this rapid growth of cities has produced. These are problems which we and the next generation face; if they can be faced quantitatively we can have more hope in the validity of their solution.

The *Research Bulletin* of the National Education Association has presented two issues recently on population: "Population Trends and Their Educational Implications,"[4] and "Schools and the 1940 Census."[5] There is also the periodical, *Population Index*,[6] and numerous governmental sources. For a local project, one could have a class attempt to answer the question, "How large a school building will we need here ten years from now?" While they cannot tell, they may come as close to the correct answer as the superintendent and school board do; and the important thing is for them to realize that such problems exist, and that estimates are likely to be better than pure guesses. A somewhat similar question would be, "Do you think our city has reached its maximum size?" Questions of this order are regularly studied by high school pupils *when we know the answers;* if we are going to prepare high school students to "take over" within a few years, we should have them study questions to which we do *not* know the answers. They will gain important insights into the difficulty of administration, government, and industry.

A specialized (though relatively undigested) source for census data is the *Abstract of the Fifteenth Census of the United States.*[7] A source for census figures which is less extensive—though normally adequate—is *The Statistical Abstract of the United States.*[8] This volume is published each year; the latest one (for 1941) contains 1940 population data, and is an almost unlimited source of figures on a large variety of topics, including both national and state data—and in some cases, data for the larger cities. The content ranges through immigration, education, climate, federal employees, finance, labor, social security, military science, the post office, power, transportation, commerce, agriculture, housing, and other phases of production and distribution. The data are of the basic sort, presented simply in tables. It is part of the statistician's function to find ways of making data interesting—and the attempt to do this can well be undertaken as a class project without great restriction by this source book as to the area in which the class wishes to work. Another fairly general source of data—though not as extensive, and possibly not as authentic, is the annual vol-

[4] January 1938, Vol. 16, No. 1, p. 1–59. Published at 1201-16th St., N.W., Washington, D. C.

[5] November 1941, Vol. 19, No. 5, p. 203–231. Available at 25 cents each.

[6] Published by the School of Public Affairs, Princeton University, and the Population Association of America.

[7] United States Bureau of the Census. *Abstract of the Fifteenth Census of the United States,* (1930). Washington, D. C.: Government Printing Office, 1933. 968 p. Price $1.50. Presumably an abstract of the Sixteenth Census (1940) will be available sometime in 1943. Meanwhile, releases are available on most any phase of the population census, directly from the Bureau of the Census.

[8] Bureau of the Census. *Statistical Abstract of the United States, 1941.* (63rd Number.) Washington, D. C.: Government Printing Office, 1942. 1017 p. Price $1.50.

ume of *The World Almanac*.[9] This *Almanac* is reasonably popular, and often gives background material in context which adds interest to the figures.

General social problems.—In addition to the reference books just listed, a number of sources present excellent material on national problems over a wide range. A series of bulletins which is now appearing is admirably adapted to statistics teaching. This series is "Problems in American Life," prepared jointly by the National Association of Secondary-School Principals and the National Council for the Social Studies. The first number was published this year, on "How Our Government Raises and Spends Money."[10] The subject is close to the interests of everyone—and undoubtedly will be for some time to come. The bulletin contains a section on teaching aids, including objectives, the construction of graphs, and an extensive bibliography. Twenty-five such bulletins, dealing with outstanding social issues, are projected. Other series readily available to teachers are the Public Affairs Pamphlets[11] of which over fifty have been printed, and the magazine *Building America*, issued eight times a year since 1935,[12] and devoted to pictures and statistics covering many aspects of American life. A quantitative analysis and history of the United States has been issued.[13] A somewhat older treatise on problems of the nation is *Recent Social Trends*.[14] An interesting project would be to locate appropriate sources for keeping its material up to date.

These several publications present statistics in relatively final or finished form—ready for the consumer, and well set in a logical context. It is worth while noting the aspects that have been selected for quantification and presentation, and how each of them has been handled.

The world situation.—In the area of international problems and the war, first mention should be made of the Headline Books,[15] by the Foreign Policy Association, for excellent, varied, and persistent graphical presentation. The University of Chicago transcripts of the *Round Table* discussions[16] usually contain significant graphs and sometimes statistics, relating to both national and international problems. The *Statesman's Year-Book*[17] gives data on all countries, and a new inter-American yearbook of statistics[18] makes additional figures on Latin America accessible. The League of Nations has

---

[9] New York *World-Telegram. The World Almanac and Book of Facts for 1942.* (Fifty-seventh year of publication.) New York: 125 Barclay Street. Issued annually. Price 70¢.

[10] National Association of Secondary-School Principals and the National Council for the Social Studies. "How Our Government Raises and Spends Money." Problems in American Life: Unit No. 1. Washington, D. C.: National Education Association, 1201 Sixteenth Street, N.W., 1942. 80 p. Price 30¢.

[11] Public Affairs Committee, 30 Rockefeller Plaza, New York, N. Y. Ten cents each; quantity discounts.

[12] Sponsored by the Society for Curriculum Study. Distributed by Americana Corporation, 2 West 45th Street, New York. Single copies, 30¢; discount on quantity orders.

[13] Louis M. Hacker, Rudolf Modley, and George R. Taylor. *The United States: A Graphic History.* New York: Modern Age Books, Inc., 1937. 243 p.

Although it is probably too mature for direct use by pupils, the teacher will find the following reference full of social and statistical interest—in part because of the unusually effective method of combining text and graphs:

Otto Neurath: *Modern Man in the Making.* New York: Alfred A. Knopf, 1939. 159 p.

[14] President's Research Committee on Social Trends. *Recent Social Trends in the United States.* New York: McGraw-Hill Book Co., 1933. 2 vol.

[15] Headline Books. Edited by Varian Fry. Published by the Foreign Policy Association, 8 West 40th Street, New York, N. Y. Some thirty titles have been published since 1935. Prevailing price, 25¢.

[16] *University of Chicago Round Table.* Published weekly; a transcript of the radio discussion given on Sunday afternoons over the National Broadcasting Company stations. Over 200 have been printed. Price, 10¢ each; quantity discounts. Subscriptions, $1.00 for six months.

[17] *Statesman's Year-Book:* Statistical and Historical Annual of the States of the World. New York: Macmillan Co. Issued annually (since 1864). The 1942 edition has 1474 pages.

[18] *Inter American Statistical Yearbook,* 1940. Edited by Raul C. Migone and others. New York: The Macmillan Co., 1940. 612 p.

issued its *Statistical Year-Book*[19] since 1927, and a number of other interesting series—particularly through its Economic Intelligence Service. Several issues of *Building America* have been devoted to international problems. It may be revealing to note how many aspects of the complex international situation can be presented in statistical terms.

Ayres' brilliantly simple yet thoroughly quantitative treatise on the first world war[20] is still to be found in libraries, and his preliminary version of preparedness in the present war[21] makes an interesting background for discussion—and conjecture as to subsequent progress. Of course no further basic statistics can be expected to be released until the close of the war, but certain series may be carried forward. Cahn publishes graphs[22] which occasionally present war expenditures, in the Chicago *Tribune* and syndicated papers. The army has prepared *A Graphic History of the War*[23] which suggests production (p. 61–63) and portrays many interesting aspects of the war. Other data are available in the economic sources which follow.

Economic trends.—Data on economic conditions are practically endless. Only a few outstanding sources will be mentioned. Most of the references already cited in previous sections include data of an economic character—particularly when economics is regarded broadly as including actual and potential human activity. Most general and comprehensive is the *Survey of Current Business*,[24] which is issued monthly, with occasional (now biennial) cumulative volumes called *Supplements* which contain over 2000 statistical series on economic matters. Many of these appear, in less detail, in the *Statistical Abstract of the United States*. A description of economic conditions in the United States about fifteen years ago can be used for background purposes. Historical series of data on the United States up to 1923 were presented by Guetter.[26] The magazine *Economic Geography*[27] is replete with graphs and data.

The automobile industry issues an annual handbook[28] which is brief, and filled with interesting data and diagrams. The latest edition portrays vividly the conversion of plants to war purposes. A number of financial institutions publish monthly bulletins, such as the Cleveland Trust Company *Business Bulletin*, which present material in attractive form, and which are often free. *Consumers' Guide*[29]

[19] League of Nations. Economic Intelligence Service. *Statistical Year-Book of the League of Nations*. New York: Columbia University Press (1932–33 through 1940–41). Earlier issues, back to 1927, were published by the World Peace Foundation, Boston. Before 1930–31 the title was: *International Statistical Year-Book* and it was prepared by the Economic and Financial Organization of the League of Nations.

[20] Leonard P. Ayres. *The War with Germany:* A Statistical Summary. Washington, D. C.: Government Printing Office. 2d ed., 1919. 154 p.

[21] Leonard P. Ayres. "The Progress of Preparedness." Cleveland, Ohio: Cleveland Trust Co., September 1940. 28 p.

[22] All of R. D. Cahn's graphs are assembled and published each year by the Chicago *Tribune* e.g., *The 1942 Chart Book*. March, 1942. 128 p. 85¢. The graphs cover many aspects of the war and various economic changes.

[23] United States War Department. *A Graphic History of the War*. September 1, 1939 to May 10, 1942. Prepared for use with the War Department Orientation Course. Washington, D. C.: Government Printing Office, 1942. 72 p. 25¢.

[24] U. S. Department of Commerce. Bureau of Foreign and Domestic Commerce. *Survey of Current Business*. Washington, D. C.: Government Printing Office. Subscription, $1.50 per year. The latest *Supplement* was for 1942. 271 p. 50¢.

[25] Conference on Unemployment, Committee on Recent Economic Changes. *Recent Economic Changes in the United States*. New York: McGraw-Hill Book Co., 1929. 2 vols.

[26] Fred J. Guetter and Albert E. McKinley. *Statistical Tables Relating to the Economic Growth of the United States*. Enlarged edition. Philadelphia: McKinley Publishing Co., 1924. 52 p.

[27] Published by Clark University, Worcester, Mass. Subscription, $5.00.

[28] Automobile Manufacturers Association. "Automobile Facts and Figures: Twenty-fourth edition, 1942." Detroit, Michigan: New Center Building, 1942. 64 p. Gratis.

[29] *Consumers' Guide*. Published by the Department of Agriculture, Washington, D. C., since 1934, monthly and semimonthly. 150,000 copies of each issue distributed free; additional copies, 5¢ each.

often publishes graphs and statistics on prices, distribution, and family expenditures, from the consumer point of view. Many local data can be provided by the Chamber of Commerce in a city, often in the form of a monthly (perhaps mimeographed) bulletin of business statistics. Also, some of the advertising bulletins prepared to attract tourists[30] afford both useful and engaging material.

Many of the sources listed contain copious references to further material. The following two sources may be cited especially for the extensive bibliographies of economic and related data which they contain.[31]

Sociological aspects.—There are other interesting statistical phases of human activity and conditions, of a more or less sociological nature. Youth surveys[32] and community surveys have been made in considerable number, and there are handbooks available for aiding in making such studies.[33] Surveys are rich in human interest and most aspects are appropriate for children of high school age to think about and even to engage in collecting data for. Cultural surveys present opportunities which can be varied to suit the local conditions and social philosophy.

Public opinion and attitudes are one important aspect of the cultural scene, and simple methods of surveying are available.[34] Reports of Gallup polls are now familiar to newspaper readers; the magazine *Fortune* also conducts and reports polls on current questions. Pupils can make various surveys of opinions and attitudes held by their own groups. It is sometimes of interest to make a census of the superstitions held.[35]

Data on various aspects of the local community may be available by virtue of special surveys having been made, such perhaps as a survey of the needs and conditions surrounding public education[36] or a survey of health conditions.[37] The annual report of the superintendent of schools will usually afford many facts. The state department of education may issue a statistical report each year or biennium. The U. S. Office of Education *Biennial Survey of Education*[38] presents data of

[30] By way of illustration: "Los Angeles, the Magic City and County." A vest pocket booklet of economic statistics. Compiled by the Research Department, Los Angeles County Chamber of Commerce. 1940. 48 p.

[31] A. W. Troelstrup. "Survey of New Pamphlet Material for Teaching Economics." In: *Economic Education* (Harold F. Clark, Editor). Chapter X, p. 140–166. Eleventh Yearbook, National Council for the Social Studies, a Department of the National Education Association. Washington, D. C.: the Council, 1940.

Rudolf Modley. *How to Use Pictorial Statistics.* "Sources of Statistics," Chapter IX, p. 89–95, and 158–166. New York: Harper & Brothers, 1937. 170 p.

[32] An outstanding example, with many graphs and human-interest data, is: Howard M. Bell, *Youth Tell Their Story.* (A Survey made for the American Youth Commission.) Washington, D. C.: American Council on Education, 1938. 273 p.

[33] M. M. Chambers and Howard M. Bell. *How to Make a Community Youth Survey.* American Council on Education Studies, Vol. III, No 2. Washington, D. C.: The Council, 1939. 45 p. 25¢.

Carl A. Jessen and H. Clifton Hutchins. *Youth: Community Surveys.* U. S. Office of Education Bulletin, 1936, No. 18-VI. Washington, D. C.: Government Printing Office, 1936. 97 p. 15¢.

Joanna C. Colcord. *Your Community.* 2d. ed. rev. New York: Russell Sage Foundation, 1941. 261 p. 85¢.

[34] "The New Science of Public Opinion Measurement." Princeton, N. J.: American Institute of Public Opinion, 1939. rev. 16 p. Gratis.

[35] Julius B. Maller and Gerhard E. Lundeen. "Sources of Superstitious Beliefs." *Journal of Educational Research* 26: 321–343; January 1933.

Otis W. Caldwell and Gerhard E. Lundeen. "Students' Attitudes Regarding Unfounded Beliefs." *Science Education* 15: 246–266; May 1931.

[36] Surveys of the local or state school system made by outside agencies usually provide many community data. Sometimes, such a survey is made by the research division of the public schools, as:

"Social Trends and the Schools." A community analysis for use in educational Planning. Newark, N. J.: Board of Education, Department of Reference and Research, March 1941. Mimeo. 82 p.

[37] *A Study of Mortality in Cincinnati for the Period 1929–1931.* Cincinnati, Ohio: Public Health Federation, December 1935. 145 p. $1.00.

[38] Published every two years since 1916–18, by the Government Printing Office, Washington, D. C. Often printed in two volumes, the second volume being tables of data, and the first volume being an interpretation of these.

great interest for the country as a whole, and certain facts for individual states and cities.

Accidents and insurance.—Life insurance, as our most mathematical business enterprise, is full of statistical opportunities. A number of the large companies publish annual reports which provide a basis for study and analysis. The Institute of Life Insurance (60 E. 42nd St., New York City) represents the large companies and has many data available. Closely connected with insurance is the subject of accidents. The National Safety Council each year publishes a highly attractive and engaging digest of accident data[39] which would fill a semester of work. The Travelers Insurance Company has issued an important series of factually based pamphlets on automobile safety.[40] When interest has been aroused it may be feasible to conduct a study of the accidents in and around the school.[41] Safety in all phases of living, including industry, is now an important topic. We lose more lives by accidents than we do by war.

Graphing

Graphing is always a subject of interest. The use of various kinds of graphic materials in school is discussed by several writers on visual education.[42] Modley, who has been the leader in popularizing pictographs in this country, has prepared several treatises on the use of graphs in school,[43] and his company[44] has issued a series of charts, the size of letter-paper, for classroom use, on many subjects of current social and scientific interest. These charts cost about six cents apiece, and are engaging both for their content and their techniques of graphing. The company is interested in cooperating with schools. The pictograph is now a popular form of graph, and is particularly well adapted to social statistics—as contrasted with the continuous curve which is particularly well adapted for scientific fields.

Background in Social Arithmetic

The statistical study of social facts can be prepared for in earlier grades by laying a background of interest and familiarity through the teaching of "social" forms of arithmetic—operations that are based upon socially significant or personally meaningful content. There will be no gain in employing such material if the social concepts or general understanding required is so far beyond the experience of the children that they cannot grasp the underlying natural relationships. There are, fortunately, teaching materials that are well adapted to the purpose.[45] We must

[39] *Accident Facts.* Chicago: National Safety Council, Inc., 20 N. Wacker Drive. Published annually in June. Over 100 pages. Price, 50¢; quantity discounts.

[40] Travelers Insurance Company, Hartford, Conn. The pamphlets are free.

[41] T. C. Holy. "Analysis of the Causes of St. Louis School Accidents." *School and Society* 30: 159–160; August 3, 1929.

L. L. Gallup and A. S. Lewerenz. "Partial Summary of Accidents to Pupils in Los Angeles Public Schools during the School Year 1929–30." *Los Angeles Educational Research Bulletin* 10: 3–7; November 1930.

Niles Anderson. "Tomorrow May Be Too Late: A Report on Accidents Involving Pittsburgh School Children." *Pittsburgh Schools* 13: 149–154; May 1939.

[42] Charles F. Hoban, Charles F. Hoban, Jr., and Samuel B. Zisman. *Visualizing the Curriculum.* "Graphic Materials," Chapter VI, pp. 211–268. New York: Cordon Co., 1937.

[43] Rudolf Modley. *How to Use Pictorial Statistics.* Chapter XV, "Pictorial Statistics in High School," pp. 145–151. New York: Harper and Brothers, 1937. 170 p.

Rudolf Modley. "Maps, Charts, Graphs, and Pictures as Aids in Economic Education." In *Economic Education* (Harold F. Clark, ed.). Chapter VIII, pp. 113–130. Eleventh Yearbook, National Council for the Social Studies, a department of the National Education Association. Washington, D. C.: the Council, 1940.

Rudolf Modley and Frank W. Hubbard. *Instructions for Chartmakers.* New York: Pictorial Statistics, 1938. 7 p. 30¢.

[44] Pictograph Corporation, 142 Lexington Avenue, New York, N. Y. A catalog of available charts will be sent upon request. Teachers will also find useful the "dictionary," *1000 Pictorial Symbols,* published by the Pictograph Corporation, New York, 1942. 57 p. $2.00.

[45] As examples, selected units from the following series may be cited:

Boyce-Beatty *Mathematics of Everyday Life Series.* Units on Finance, Health, and Leisure.

recognize that the presumed maturity of understanding and interest on the part of the senior high school pupil does not come about automatically and that introductory approaches to the quantitative understanding of various social problems are of genuine importance in bringing about the maturity desired.

A number of high school textbooks in mathematics include short sections on statistics.[46] Such texts, however, usually treat statistics as just a more or less popular section of regular mathematics.[47] No high school text that the writer has seen gives recognition to the point that statistics is of greater potential utility to the majority of high school pupils than the conventional mathematics is. All high school pupils become citizens; only a portion enter vocations calling for ability in algebra, geometry, and trigonometry. Most fortunate, of course, are those students who can have training in both the social and scientific types of mathematics; but the immediate need is for the development of statistics courses.

Concluding Statement

We know that every status and every change has its quantitative aspects. To ignore these aspects is a mistake. To concentrate attention on them exclusively is likewise a mistake. A course in high school statistics should not become either a purely social studies course or a purely mathematics course, but it should be that combination of the two which represents and emphasizes statistical data and techniques as a normal part of thinking about problems which are important in our personal and civic lives.

We must not think of statistics as a set of devices or procedures. Statistics at its best is a point of view. It is an outlook on life which holds that quantitative aspects, quantitative comparisons, and systematic quantitative relations are usually of considerable importance. It is a frame of thinking which demands knowledge of important quantitative elements before a conclusion is attempted. It develops an expectancy which is not satisfied by generalizations which lack at least some semblance of quantitative modification. It creates an attitude of mind which invariably queries, In what degree is this true? For what proportion of the cases does it hold? How widely do different cases differ in this respect? How stable is the tendency from group to group or time to time? Under what conditions is the generalization true?

To objectify such questions and permit answers to them, statistics utilizes the frequency distribution and employs various methods for analyzing distributions; but it is the well directed quantitative questions rather than the methods of calculation which constitute the fundamentally important contribution to general education. In problem solving *the right answer is never so important as the right question.* For without an illuminating question, what does the answer matter?

Published by the Inor Publishing Co., 207 Fourth Avenue, New York.

Raleigh Schorling and John R. Clark. *Mathematics in Life.* Yonkers-on-Hudson, New York: World Book Co., 1937. 437 p.

Guy T. Buswell, William A. Brownell, and Lenore John. *Daily-Life Arithmetics,* Book Three. Boston: Ginn and Co., 1938.

William Betz. *Basic Mathematics.* Boston: Ginn and Co., 1942. 502 p.

G. M. Ruch, F. B. Knight, and J. W. Studebaker. *Mathematics and Life.* (Standard Service Series.) Books I–III. Chicago: Scott, Foresman and Co., 1937.

See also the following discussion: Harry E. Benz, "Arithmetic in the Senior High School." In Arithmetic in General Education, Chap. 6, pp. 119–139. Sixteenth Yearbook, National Council of Teachers of Mathematics. New York: Teachers College, Columbia University, 1941.

[46] Probably the best of these is: U. G. Mitchell and Helen M. Walker. *Algebra: A Way of Thinking.* Chapter XI, "Statistics," pp. 211–282. New York: Harcourt, Brace and Co., 1936.

[47] Drake reported experience with a unit on statistics in the ninth grade, but he appears to have been primarily concerned with the computational side of statistics. He closed his report with the comment: "Possible improvements lie in the introduction of more social data." Richard M. Drake. "Statistics for Ninth Grade Pupils." *Mathematics Teacher* 34: 16–22; January 1941.

We should therefore reverse the normal emphasis upon correctness in mechanical calculation, more or less without purpose to the student, and dwell upon the *setting* out of which fruitful questions should arise. In teaching we should naturally include various rudimentary forms of calculation, to aid the student in satisfying his growing insistence upon quantitative answers. But our emphasis will not be upon the answers, nor upon the methods of arriving at them, but rather upon that dissatisfied state of mind which should prevail until one has been supplied with quantities which will permit significant answers to be found.

Training in accordance with the emphasis which has been discussed would not make research workers. It would not directly produce statisticians. But it would provide a dynamic background of quantitative experiences closely connected with urges and concerns which are vital to the individual; and rooted in that rich background, the high school students would find basic orientations that would prove of great service in their later lives—whether in college, vocational, civic, or personal activities. Statistics offers a quantitative approach to problems in social areas which can become as effective in this field as the accustomed mathematical approach to problems in physical areas has proved to be. I look upon the teaching of statistics in high school as providing the first direct step in making that approach to social problems more natural and more habitual in the lives of millions of future adult citizens.

The Tables Are Bare

Multiplication a la Mother Goose

By LOWRY W. HARDING *and* HELEN E. HUTCHINSON
The Ohio State University

The schoolroom was hot, the air was still,
  In grade Three B, in old Jonesville.
Now Jonesville was known as a provincial town,
  Where habit and custom were hallowed ground.

And so it was settled with an iron hand,
  That Miss Jenny Winkle drove her small band.
On Reading and writing, in Geography too,
  She forced her pupils to worry and stew.

But 'Rithmetic was the hardest to learn,
  For every child seemed that subject to spurn.
Thirty small grimy faces were wrinkled and frowned,
  'Til all the examples were correctly set down.

The children then breathed big sighs of relief,
  Not knowing how soon they'd come to grief.
Miss Winkle soon said, with a solemn air,
  "We'll start on our *tables*, so have a care!"

"John, you may begin with the table of two's.
  "Hurry and say them, or Mary I'll choose!"
"Two one's are—uh—two; two two's are, er, four;
  "Two three's are—uh—, don't know anymore!"

"What? Then open your book and study hard that part!
  "You stay after school and learn it by heart!"
So John stayed after school 'til 'twas almost night.
  —Never learned the tables—just exactly right!

Just memory alone is really not enough,
  Learning also requires—understanding, and stuff!
Psychologically speaking, "The Tables Are Bare,"
  Of meaningful experiences to make a child *care*.

You may lead a horse to water, but you can't make him drink.
  That saying about horses is to learning—theory a link.
Thus the moral of this story is clear in a wink,
  Don't cram with "knowledge," but help children *Think!*

◆ THE ART OF TEACHING ◆

Developing Reflective Thinking Through Geometry

By INEZ M. COOK
Lincoln High School, Lincoln, Nebraska

MATHEMATICS TEACHERS have long maintained that one of the principal contributions of demonstrative geometry to general education is the development of logical reasoning. Within recent years there has been a growing tendency on the part of educators to question the outcome of the usual geometry classroom procedure. They are seeking evidence that pupils, after devoting a year to such study, are familiar with the nature of deductive thought and that they can and do use that same type of thinking in situations outside of geometry.

Teachers of geometry should be among the first to make a critical examination of the results of their teaching. One criterion for judging the effectiveness of a method of teaching geometry is the extent to which the reasoning ability developed by the subject is applied to outside situations demanding the same kind of reflective thinking. The teacher should consider whether the teaching techniques employed have led pupils to think more clearly, to weigh evidence critically, to accept conclusions cautiously, and to discover and prove relationships, not only in geometry but also in everyday situations involving scientific thinking.

In attempting to meet this challenge, a group of mathematics teachers at Lincoln High School, Lincoln, Nebraska, felt that improvement might be effected by reorganizing their classroom procedure along the following lines:

1. Introduce, through the use of everyday situations, the reasoning patterns necessary for an understanding and mastery of the geometry content.

2. As the geometry situations become more complex, parallel them, so far as possible, with like situations from non-geometric material, interweaving the two so that each is an aid in understanding the other.

3. Make the transfer of training a conscious and continuous procedure—from geometric to non-geometric problems and vice versa.

4. Through the use of many and varied types of materials help pupils to recognize the reasoning patterns of geometry in everyday life situations and make them a part of their everyday thinking.

Before attempting these changes in classroom procedure, a committee of four teachers worked together for one semester reading and collecting data on experiments and progress others have made in similar revisions of geometry teaching. This material was summarized, discussed with the other mathematics teachers and made available for future reference. The next semester each member of the committee working with a group of beginning geometry pupils, began the development of a teaching technique which would include the points mentioned above. Weekly group conferences gave these teachers an opportunity to discuss their problems and make plans for further progress. One of the greatest difficulties they experienced was the lack of suitable non-geometric material. However, at the close of the semester these teachers felt some progress had been made and they were willing to share their material collected and techniques with others in the department who were interested.

With the opening of school in September of 1941, four more teachers wished to join in the experimental work, making eight teachers who were including non-geometric material in their work and revising their procedure. Three teachers were continuing the usual or traditional classroom procedure. This situation provided an experimental group and a control group. In order to determine the results of this change in method of teaching a testing program was set up. It included scientific reasoning tests and geometry achievement tests.

A series of five tests on different types of reasoning was given to all beginning geometry classes during the first week of the semester and a similar series at the close of the semester. For their advice and cooperation in supplying material, the geometry committee is indebted to Dr. Ralph Tyler of the University of Chicago and to Mr. Ray G. Wood, of the State Department of Education of Columbus, Ohio. Since there seemed to be, on the tenth grade level, no standardized test available which included the various types of reasoning situations desired, the geometry committee compiled two forms of tests which contained similar items of comparable difficulty. The following types of reasoning situations were included in each of the two forms:

Test I—Syllogisms and indirect seasoning
Test II—Underlying assumptions
Test III—Drawing conclusions from printed matter
Test IV—Making interpretations from a graph and deciding on best and poorest solutions to a problem
Test V—Drawing a conclusion and selecting supporting statements

In the present emergency when the need for mathematical training is felt so keenly it is of utmost importance to consider the effect any change in classroom procedure might have on mathematical achievement. Accordingly at the close of the semester a geometry achievement test was given to all beginning geometry classes. Suggestions as to types of exercises which might well be included in such a comprehensive test were obtained from an examination of many standardized tests for geometry. Using the geometry content material covered by all classes as a basis, the committee made a test which included six parts and required two sixty-five minute class periods to administer. The test was intended to measure ability to do reflective thinking in geometric situations and the mastery of geometry content. The weighting of the various items in the test was the result of the combined judgment of the committee. The total possible score was 193 points. To insure uniformity in scoring papers, one teacher scored the same part on all papers.

Before making an analysis of the results of this testing program, the scores of all pupils who did not take all three tests were discarded. This left 326 pupils in the experimental group and 76 pupils in the traditional or control group.

At the opening of the semester no attempt was made to classify pupils in order that the two groups would be of the same ability or previous training. Except for the few who might be repeating the course, none had any previous training in formal geometry. The I.Q. was used as a measure of ability. The following table shows these findings:

	I.Q. Rating	
	Traditional	Experimental
No. of pupils	76	326
Q_3	124.5	124.4
M	117.0	116.2
Q_1	108.5	106.9
Range	95 to 160	86 to 168

These findings apparently justify the conclusion that, using the I.Q. as a

measure of ability, the two groups were comparable and any differences in testing results were probably not due to differences in general ability. The summary of scores in both September and January reasoning tests is given in the following table:

	Reasoning Tests			
	September		January	
	Traditional	Experimental	Traditional	Experimental
No. of pupils	76	326	76	326
Q_3	78.0	76.3	94.0	96.5
M	66.8	67.2	84.5	90.4
Q_1	59.0	59.9	76.5	82.4
Range	47 to 96	28 to 106	60 to 100	52 to 106

The total possible score in each of these tests was 120 points. The results of the September testing show that the median of the two groups varied by only a fraction of a point, the upper and lower quartiles showing a slightly wider variation, one favoring the traditional group and the other the experimental. In general, using these tests as a measure, the two groups were found to be almost equal in reasoning ability in September.

In January the median of the traditional group shows an increase of 17.7 points while the experimental group had made an increase of 23.2 points, a gain of 5.5 points, or a 31% greater gain in reasoning ability in non-geometric situations than did the traditional group. Similarly the upper quartile showed a 25% and the lower quartile a 29% greater improvement than that shown by the traditional group.

Using the median of the traditional group as a basis, it is found that 50% of the traditional group was exceeded by 51.5% of the experimental group in September while in January 50% of the traditional group was exceeded by 70.5% of the experimental group.

The results of the geometry achievement test are tabulated below:

Geometry Achievement Test		
No. of pupils	Traditional	Experimental
	76	326
Q_3	150.0	156.9
M	136.5	137.8
Q_1	105.5	109.3
Range	58 to 179	38 to 193

The highest possible score in this test was 193 points.

It is noted here that the experimental group made a slightly higher median than the traditional group, and that both the Q_1 and Q_3 points are noticeably higher. With the median of the traditional group as a basis it is found that 50% of the traditional group was exceeded by 50.9% of the experimental group.

At the close of the semester each teacher of experimental classes was asked to make an estimate of the amount of time devoted to non-geometric material. Six teachers estimated approximately one-fourth of the time, one teacher slightly more and the other slightly less. The above table indicates that, with approximately one-fourth the time devoted to non-geometric material, the experimental groups were able to do slightly better work in geometry than those groups devoting the entire time to geometry material alone.

Some conclusions reached as a result of this experiment are as follows:

1. It would be unwise to make any estimate of the possibilities of functional geometry teaching from this one semester of experimental work. It should be kept in mind that none of the eight teachers of the experimental group had had any special training in this method of teaching; however all were teachers experienced in presenting geometry by the traditional method.

2. Teachers of the experimental classes were much encouraged by the results of

the testing program but were far from satisfied. They believed that with more experience in the use of non-geometric material the gain both in reasoning ability and in geometry achievement would probably be greater. However, these results strengthened their belief that they were moving in the right direction.

3. Using the I.Q. as a measure of general ability and the reasoning test as a measure of scientific reasoning, any difference in the two groups at the beginning of the semester was negligible.

4. At the close of the semester it was found that both groups had made considerable gain in their ability to reason in non-geometric situations, but the experimental group showed a gain about 30% greater than that made by the control group. The results at the beginning of the semester showed that 50% of the control group were exceeded in scientific reasoning by 51.5% of the experimental group but at the close of the semester 50% of the control group were exceeded by 70.5% of the experimental group.

5. Pupils in the experimental group, while devoting only about three-fourths as much time to geometry material, were able to do slightly better in geometry achievement than the group devoting all the time to geometric material.

Regional Meeting
National Council of Teachers of Mathematics

Saturday March 6, 1943

La Salle Hotel, Chicago, Illinois

MORNING SESSION: 9:30 A.M., C. M. Austin, presiding

I. The A, B and C Programs for Training in Meteorology in the Army Air Forces
Prof. William L. Hart, Consultant Army Air Forces Academic Committee for Premeteorological Training.

II. What Does the Industrialist Expect of the Graduate of our Non-technical High School in the Way of Mathematical Training?
John T. Riddell.

Luncheon: 12:00

Showing of Mathematical Films

AFTERNOON SESSION: 2:00 P.M., Miss Martha Hildebrandt, presiding

I. Report of the National Emergency Committee on Mathematics.
Rolland R. Smith, Pres. of National Council.

II. Report on Chicago Emergency Committee on Mathematics.
O. F. Fowler, District Supt., Chicago High Schools.

III. What Should We Be Doing in the Classroom?
Round Table Discussion.
Speakers to be announced later.

Salute Lieutenant Brown !

KENNETH BROWN, who for some time has been chairman of State Representatives of The National Council of Teachers of Mathematics, has enlisted in the Navy and has been commissioned as a Senior Lieutenant. Mr. Brown has been a very faithful worker in the Council and, as a result of his untiring efforts, the Council's interests have been improved and its publications have received wider acceptance. Until Lieutenant Brown's successor has been appointed, the various State Representatives should communicate with the Editor about anything concerning the work of the Council in their respective states.

W. D. R.

EDITORIALS

How Popular Is Mathematics Among High School Pupils?

Fortune magazine recently made a survey of Public Opinion* which is "A two-part self-portrait of American high school youth . . . what it thinks of itself, its country and its future." What these boys and girls thought of the subjects they had been studying in school will be of interest to the teachers of the various subjects represented. What they thought of mathematics will be of particular interest and profit to the readers of THE MATHEMATICS TEACHER.

The following table gives the composite answer to the question:

Of all the subjects you have taken so far in high school; which one have you liked best? Liked least?

	best	least	
Mathematics—algebra, geometry, trigonometry	20.0%	26.7%	Mathematics—algebra, geometry, trigonometry
English—grammar, composition, literature, etc.	17.7	22.2	English—grammar, composition, literature, etc.
Sciences—general science, biology, chemistry, other (except social)	14.8	13.4	Languages—French, Spanish, Latin, etc.
History	11.0	12.4	History
Vocational courses—home economics, typing, other business	10.3	8.7	Sciences—general science, biology, chemistry, other (except social)
Languages—French, Spanish, Latin, etc.	6.8	3.9	Vocational courses—home economics, typing, other business
Civics, government, social science, etc.	1.8	3.2	Civics, government, social science, etc.
Don't know	2.4	5.3	Don't know

In commenting on the above table *Fortune* goes on to say:

There's no mystery about the fact that English and mathematics rank, respectively, first and second as most liked *and* most disliked. In the first place they are the most universally given, and run generally through all four years of school. But cross tabulation shows very clearly that there are two kinds of students whose preferences form consistent patterns. Those disliking English, languages, and history are devoted to mathematics and the laboratory sciences; and vice versa. For example:

	Students liking the following least—			
Like the following subjects best, by these percentages—	English	History	Mathematics	Sciences
Mathematics, biology, chemistry, other sciences	51.9%	41.8%	14.6%	24.6%
English, languages, history	15.6	22.2	57.3	47.5
Other	30.3	33.7	26.5	27.4
Don't know	2.2	2.3	1.6	.5

One further observation is to be made from analysis of the answers: there are almost no differences in preference between the students on the basis of the amount of information they have on public affairs. This may be considered as still further evidence of the dissociation between school work and general knowledge.

Here is a challenge for all teachers of mathematics. When pupils like mathematics they like it. When they hate mathematics they hate it. Why this great gap? There is strong reason for believing that if *Fortune* could also have obtained reasons for the pupils' likes and dislikes of mathematics, they would have discovered that when pupils like their teachers they generally like the subjects and vice versa. A common reply to the question "Why do you like mathematics?" is, "I like my teacher." Similarly, a pupil who does not like mathematics often gives as a reason, "I do not like the teacher."

W. D. R.

* Reprints (no charge) of this material from the November and December (1942) issues may be secured by writing to the General Manager, *Fortune* Magazine, Time and Life Building, Rockefeller Center, New York City.

IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

The American Mathematical Monthly
Vol. 49, No. 6, June–July 1942.

1. Griffin, F. L., "Undergraduate Mathematical Research," pp. 379–385.
2. Jackson, Dunham, "A Comment on Differentials," p. 389.
3. Church, Alonzo, "Differentials," pp. 389–392.
4. Whitman, E. A., "The Film–'A Triple Integral'", pp. 399–400.

Vol. 49, No. 9, November 1942.

1. Bell, E. T., "Newton after Three Centuries," 553–575.
2. Fu Traing Wang and Chuan-Chih Hsiung, "A Theorem on the Tangram," pp. 596–599.
3. Curtis, H. B., "Note on Integrating Factors," pp. 610–611.
4. Lenser, W. T., "A Note on Semi-Logarithmic Graphs," pp. 611–613.
5. "Occupational Classification of Mathematicians," pp. 624–630.

School Science and Mathematics
December 1942, Vol. 42, No. 9.

1. Reod, Cecil B., "Selection of Mathematics Texts," pp. 809–812.
2. Eckel, Arthur F., "The Computogram," pp. 827–830.
3. Winsey, A. Reid and Sands, Lester B., "Contributions of Mathematics in the Development of Art," pp. 845–852.
4. Nyberg, Joseph A., "Notes from a Mathematics Classroom," (continued) pp. 865–868.
5. Struyk, Adrian, "The Solution of the Quadratic Equation," pp. 882–883.
6. Syer, Henry W., "The Uses of Mathematics by the Other Sciences," pp. 884–887.
7. McGee, Joseph A., "Behind the Physics Problem," pp. 887–889.
8. Storm, Robert, "Mathematics in War," pp. 891–892.
9. Barker, Catherine Ann, "Seventh Grade Mathematics Yuletide Greeting," pp. 890–891.

Miscellaneous

1. Brownell, W. A. and Grossnickel, F. E., "Teaching of Mathematics in Grades I through VI," *Review of Educational Research*, 12: 386–404, October 1942.
2. Burnett, R. W., "Teacher Education in Science and Mathematics," *Review of Educational Research*, 12: 443–450, October 1942.
3. Buswell, G. T., "Selected References on Elementary School Instruction; Arithmetic," *Elementary School Journal*, 42: 212–15, November 1941; 43: 174–175, November 1942.
4. Carper, D., "Seeing Numbers as Groups in Primary-Grade Arithmetic," *Elementary School Journal*, 43: 166–170, November 1942.
5. Christofferson, R. O. and Wittich, W. A., "Navy Statistics on Mathematics Retention—a Challenge to the School," *School and Society*, 56: 502–504, November 21, 1942.
6. Fitzsimmons, T. A., "Mathematics, an Elementary Stumbling Block," *National Schools*, 30: 41, November 1942.
7. Gregory, L. G., "Teacher Preparation in the Field of Arithmetic," *Sierra Educational News*, 38: 20, November 1942.
8. Hartung, M. L., "Teaching of Mathematics in Senior High School and Junior College," *Review of Educational Research*, 12: 425–434, October 1942.
9. Jones, E. M. C., "Projects in Mathematics," *School* (Secondary Edition), 31: 229–232, November 1942.
10. Kier, T. L., "What Method of Subtraction Shall We Teach?" *Nebraska Educational Journal*, 22: 256, November 1942.
11. "Natural Sciences and Mathematics," *Review of Educational Research*, 12: 359–452, October 1942.
12. Owens, C. G., "Christmas Project in Mathematics," *Ohio Schools*, 20: 384, November 1942.
13. Robertson, M. L., "Teaching of Science and Mathematics in Extra School Education," *Review of Educational Research*, 12: 435–442, October 1942.
14. Trusler, J. W., "Arithmetic Program: Grades I to IV," *School* (Elementary Edition) 31: 235–238, November 1942.

NEWS NOTES

The Women's Mathematics Club of Chicago and Vicinity held its first meeting of the school year on Oct. 3 at the Georgian Hotel in Evanston. Following a luncheon, Dr. H. T. Davis, Professor of Mathematics at Northwestern University and author of the new book, *Alexandria, The Golden City* gave a delightful talk entitled "Luncheon with Euclid." Miss Mary Jane Hartman of Oak Park High School, the new president, presided and introduced the other officers: Vice-Pres., Mrs. Doleis of Amundsen High School; Sec., Miss M. Eccles, Kelly High School; and Treas., Miss Virginia Terhune of Proviso Township High School.

Dr. John R. Clark of Teachers College Columbia University, was the speaker at the recent dinner of the Mathematics Section of the Maryland State Teachers Association. His topic was "Secondary School Mathematics in the Crisis."

The Association of Mathematics Teachers of New Jersey held its Seventy-fifth Regular Meeting in Kirkpatrick Chapel, Rutgers University, New Brunswick, New Jersey, on Saturday, May 2, 1942. The program follows:

General Theme: Mathematics in a Victory Program.

1. Mathematics for Nursing—C. B. Bogart, Jr., Instructor in Mathematics for Nurses at the Muhlenburg Hospital, Plainfield, N. J.
2. Mathematical Education in Wartime—Presidential address by Dr. Howard F. Fehr, State Teachers College, Montclair, N. J.
3. Discussion and comments from the floor.

Annual Business Meeting.

Election of Officers for 1942-43.

In the presentation of his topic, Mr. Bogart convincingly demonstrated the vital importance to nurse training of a thorough knowledge and a 100% mastery of such skills and concepts as are involved in the use and interpretation of fractions, decimals, ratios, per cents, proportions, equations, formulas, graphs, and the three systems of measurement—Metric, Apothecary, and Household. He then submitted test results covering these skills—results obtained from a recent testing program administered by him to the student nurses under his instruction. These results disclosed a very apparent weakness in the acquirement of fundamentals of mathematics education; they also presented to all levels of elementary and secondary schools a renewed challenge for a clarification of concepts, and for greater emphasis upon meaningful learning experiences.

Dr. Fehr's presidential address was most enthusiastically received as being at once timely and practical. His own summarization of findings and recommendations follows:

"To summarize:—The Army, Navy, Industry, and Scientific Organizations indicate that the mathematics teachers can be of immediate aid in the war emergency by securing the following:

"1. Every boy in high school should study mathematics according to his abilities, the more capable taking four years of mathematics including trigonometry and solid geometry.

"2. Advisors must realize the acute need for mathematics and direct all capable boys into such courses.

"3. It is not necessary to create new courses or to entirely re-organize the present courses in the wartime effort. If our objectives are valid, and our methods of teaching will obtain these objectives, all that is needed is an added emphasis on the practical.

"4. Motivation can be provided through the use of simple formulas, scale drawing, and geometric theorems directly involved in military tactics. The teacher should know these applications.

"5. Good teaching of straight mathematics by fully qualified teachers is what is needed. Such teaching is a far superior procedure than to attempt courses in aeronautics or navigation in the high school, given either by teachers not fully qualified to teach them, or taken by pupils who have not the necessary foundations for their successful study.

"6. Modify your subject matter and your rate of teaching to attain complete understanding, and you will best fit boys and girls for both peacetime and wartime uses of mathematics.

"7. For the present juniors and seniors in high school, who have not had the regular mathematics, give an abbreviated course immediately, similar to that suggested by the Army Aviation Corps.

"A centipede was walking gayly along the road when a frog in the ditch below cried, 'Say,

Mr. Centipede, with all those legs, how do you know with which one to start first?' The centipede stopped, thought, and said to himself, 'Which leg do I start with first?' He tried one leg, then another, and soon his legs became so twisted that he fell over and slid down the bank into the ditch beside the frog who looked upon him with pity. The centipede kept thinking, 'Which leg do I start with first?' Suddenly, the centipede exclaimed, 'I've got it! Now I know with which leg I start first. *Now I know how I walk.*' With this enlightened frame of mind, the centipede gradually untangled his legs, climbed up out of the ditch, and went on his way, faster, more gracefully, and with greater ease than he had ever gone before.

"Let us send our students away in the same frame of Mathematical Mind."

Association of Mathematics Teachers of New Jersey

Officers for 1942–43.

President—Mrs. Florence Gorgens, Jefferson High School, Elizabeth, New Jersey.

Vice-Presidents—Hubert B. Risinger, Vernon L. Davey Junior High School, East Orange, New Jersey; Dr. D. R. Davis, State Teachers College, Montclair, N. J.; Dr. Fred L. Bedford, State Teachers College, Jersey City, N. J.

Secretary-Treasurer—Mary C. Rogers, Roosevelt Junior High School, Westfield, N. J.

Corresponding Secretary—Mrs. Dorothy Frapwell, Lafayette Junior High School, Elizabeth, N. J.

Recording Secretary—Margaret M. Dunn, Junior High School, Bloomfield, N. J.

The first meeting of the Men's Mathematics Club of Chicago and Metropolitan Area was held on Friday, October 16, 1942. The guest speaker was Dr. H. T. Davis of Northwestern University, who spoke on the topic "Dinner with Archimedes."

Miss Ruthella Moseley, 929 N. 4th Street of Atchinson, Kansas, writes that "the October 1942 issue of The Mathematics Teacher is excellent. I wish every school administrator would read it."

Harvard Accelerates Its Program of Teacher Education

The degree of Master of Arts in Teaching established at Harvard in 1936 has now been reorganized. Taking account of undergraduate studies in Education, the program now makes it fully possible for competent students from good colleges to attain the degree after one year of graduate study at Harvard. By a rearrangement of the material covered in fundamental courses in Education (Educational Psychology, Educational Measurement, Philosophy of Education, Secondary Education, and Principles of Teaching), this part of the program can be covered in twenty-four semester hours, or three-quarters of a full year of graduate work. The other quarter of the year will be devoted to advanced work in the subject to be taught. Students who have had good training in Education as undergraduates may devote more than a quarter of their graduate year to the study of their subject or a related field.

Undergraduates in Harvard College and Radcliffe College may now enter an integrated five-year program leading to the Bachelor's degree at the end of the fourth year and the A.M. in Teaching at the end of the fifth year. Decision to enter this program must be made at the close of the sophomore year. Nine semester hours in Education will be taken in the junior and senior years; sixteen semester hours in Education in the graduate year. Fields of concentration (majors) have been recognized in the Teaching of the Classics, English, Fine Arts, Mathematics, Modern Foreign Languages, Music, the Natural Sciences, or the Social Sciences.

These changes constitute a somewhat radical departure from the former Harvard program for the education of teachers. No change of principle or aim is involved, but the work has been brought together into a more coherent scheme and the degree has been made much more accessible. The program is still under the joint direction of the Faculty of Arts and Sciences and the Faculty of Education. President James Bryant Conant is Chairman of the Administrative Board under which the work is conducted. Professor Henry W. Holmes, formerly Dean of the Graduate School of Education and now Chairman of the Committee on Educational Relations, is Secretary of the Board. A pamphlet concerning the new program for the degree of Master of Arts in Teaching is available on request.

Army Air Force Offers New Meteorological Training

Mastery of the Science of Meteorology requires proficiency in and thorough knowledge of higher mathematics and physics. To train Weather Officers in these and in other related subjects, a threefold educational program has

been devised, open to young men of America, possessing good health and scientific aptitudes. The length of the course of studies will vary according to the academic background of the individual.

Under plan "A," or Advanced Meteorology, open to men of 18 to 30 years inclusive, a student who has completed two years of college, with higher mathematics through differential and integral calculus and one full year of college physics, will receive a further eight months' training in topics dealing with weather and with weather forecasting.

Under plan "B," or Premeteorology, open to men of 18 to 30 years inclusive, and of six months' duration, is available to students having completed one year of college with mathematics through college algebra, trigonometry, and analytic geometry.

Plan "C," or Basic Premeteorology, twelve months in length, is designed for the high school graduate, 18 to 21 years inclusive who has mastered algebra and plain geometry, and has completed one year of high school science.

Both "B" and "C" programs prepare students for entry into the "A" course.

In each of the three meteorology courses, accepted applicants are enlisted in the Army Air Forces, receive Army pay, uniforms and living allowances. Upon successful completion of the advanced course, students are eligible for commissions as Second Lieutenants in the Air Corps and will be placed in the field to operate weather stations at our many air bases.

It is the opinion of many educators that this is the most unusual educational opportunity ever offered to American Youth. All subjects in the three programs are given at the college level and credit toward an academic degree will be granted at all of the participating twenty-nine geographically distributed colleges and universities. Even more noteworthy is the vital character of the work to be performed at the conclusion of the training. Most, if not all, of the graduated weather officers will be moved to combat zones, to serve as central cogs in our war machine.

Both civilians and enlisted men may apply if qualified. The full particulars concerning the Meteorology training may be had by writing to:

"Weather"
Chicago, Illinois

The scientific emphasis of the program has contributed materially to the existing demand for capable instructors of mathematics and physics. Men with teaching experience in these subjects are requested to communicate with Dr. Carl G. Rossby, Head of the Meteorological Institute of the University of Chicago to obtain full information on teaching openings.

UT non-finitam Seriem finita coërcet,
Summula, & in nullo limite limes adeſt:
Sic modico immenſi veſtigia Numinis hærent
Corpore, & anguſto limite limes abeſt.
Cernere in immenſo parvum, dic, quanta voluptas!
In parvo immenſum cernere, quanta, Deum!

Jacques Bernoulli in "Ars Conjectandi," 1713

Even as the finite encloses an infinite series
And in the unlimited limits appear,
So the soul of immensity dwells in minutia
And in narrowest limits no limits inhere.
What joy to discern the minute in infinity!
The vast to perceive in the small, what divinity!

Translated by Helen M. Walker

New Books Received

Aviation Education Research Group, Teachers College, Columbia University and University of Nebraska, ELEMENTS OF PRE-FLIGHT AERONAUTICS FOR HIGH SCHOOLS, The Macmillan Company, 1942. Pp. 556.

Aviation Education Research Group, Teachers College, Columbia University and University of Nebraska, Teachers Manual for ELEMENTS OF PRE-FLIGHT AERONAUTICS FOR HIGH SCHOOLS, The Macmillan Company, 1942. Pp. 113.

Aviation Education Research Group, Teachers College, Columbia University and University of Nebraska, SCIENCE OF PRE-FLIGHT AERONAUTICS FOR HIGH SCHOOLS, (Air Age Education Series), The Macmillan Company, 1942. Pp. 868.

Aviation Education Research Group, Teachers College, Columbia University and University of Nebraska, Teachers Manual for SCIENCE OF PRE-FLIGHT AERONAUTICS FOR HIGH SCHOOLS, The Macmillan Company, 1942. Pp. 248.

William Betz, BASIC MATHEMATICS, Ginn and Company, 1942. pp. 502. Price $1.48.

Raymon W. Brink, SPHERICAL TRIGONOMETRY, D. Appleton-Century Company, 1942. Pp. 62. Price $.75.

George L. Bush, SCIENCE EDUCATION IN CONSUMER BUYING, Bureau of Publications, Teachers College, Columbia University, 1941. Pp. 227. Price $2.35.

Harold G. Campbell and F. Lynwood Wren, DISCOVERING NUMBERS, D. C. Heath and Company, 1942. Pp. 280. Price $.80.

Harold G. Campbell and F. Lynwood Wren, EXPLORING NUMBERS, D. C. Heath and Company, 1942. Pp. 264. Price $.80.

Harold G. Campbell and F. Lynwood Wren, NUMBER ACTIVITIES, D. C. Heath and Company, 1942. Pp. 247. Price $.80.

Harold G. Campbell and F. Lynwood Wren, NUMBER EXPERIENCES, D. C. Heath and Company, 1942. Pp. 248. Price $.80.

Robert Coleman, Jr., THE DEVELOPMENT OF INFORMAL GEOMETRY, Bureau of Publications, Teachers College, Columbia University, 1942. Pp. 178.

Nelson M. Cooke, MATHEMATICS FOR ELECTRICIANS AND RADIOMEN, McGraw-Hill Book Company, 1942. Pp. 604.

Hollis R. Cooley, Palmer H. Graham, Frederick W. John, Arthur Tilley, LOGARITHMS, TRIGONOMETRY, STATISTICS, McGraw-Hill Book Company, 1942. Pp. 280. Price $2.00.

John C. Cooley, A PRIMER OF FORMAL LOGIC, The Macmillan Company, 1942. Pp. 378. Price $3.00.

W. Ben Dyess and Robert O. Gilmore, MATHEMATICS OF BUSINESS AND FINANCE, McGraw-Hill Book Company, 1942. Pp. 214. Price $4.00.

Lt. W. C. Eddy, A. H. Brolly, Ensign E. S. Pulliam, E. C. Upton, G. W. Thomas, WARTIME REFRESHER IN FUNDAMENTAL MATHEMATICS, Prentice-Hall, 1942. Pp. 248. Price $1.05.

Paul L. Evans, MATHEMATICS FOR TECHNICAL TRAINING, ALGEBRA, Ginn and Company, 1942. Pp. 126. Price $1.25.

Paul L. Evans, MATHEMATICS FOR TECHNICAL TRAINING, CALCULUS, Ginn and Company, 1942. Pp. 126. Price $1.25.

Paul L. Evans, MATHEMATICS FOR TECHNICAL TRAINING, TRIGONOMETRY, Ginn and Company, 1942. Pp. 192. Price $1.25.

Vernor C. Finch, Glenn T. Trewartha, M. H. Shearer, Frederick L. Caudle, ELEMENTARY METEOROLOGY, McGraw-Hill Book Company, 1942. Pp. 301. Price $1.76.

Ferdinand Fischer and James A. Clear. DESCRIPTIVE GEOMETRY. The Bruce Publishing Company, 1942. Pp. 242. Price $3.00.

Ruth Wood Gavian, EDUCATION FOR ECONOMIC COMPETENCE IN GRADES I TO VI, Bureau of Publications, Teachers College, Columbia University, 1942. Pp. 190. Price $2.10.

William A. Granville, Percey F. Smith and James Mikesh, SPHERICAL TRIGONOMETRY, Ginn and Company, 1942. Pp. 313. Price $1.25.

Edward K. Hankin, MERRIMAN'S STRENGTH OF MATERIALS, John Wiley & Sons, 1942. Price $1.50.

Walter W. Hart, BASIC MATHEMATICS, D. C. Heath & Company, 1942. Pp. 456.

Walter W. Hart, Cottell Gregory and Veryl Schult, MATHEMATICS IN DAILY USE, D. C. Heath & Company, 1942. Pp. 376. Price $1.32.

Walter W. Hart and William L. Hart, PLANE TRIGONOMETRY, SOLID GEOMETRY AND SPHERICAL TRIGONOMETRY, D. C. Heath and Company, 1942. Pp. 124. Price $2.60.

William L. Hart, PLANE TRIGONOMETRY WITH

APPLICATIONS, D. C. Heath and Company, 1942. Pp. 124. Price $2.00.

Miles C. Hartley, TRIGONOMETRY (Plane and Spherical), The Odyssey Press, 1942. Pp. 322. Price $1.60.

Herbert E. Hawkes, William A. Luby and Frank Touton, FIRST-YEAR ALGEBRA, New Edition, Ginn and Company, 1942. Pp. 495. Price $1.40.

B. C. HAYNES, METEOROLOGY FOR PILOTS, Civil Aeronautics Bulletin No. 25, Sept. 1940, Pp. 167. Price $1.25.

B. C. Haynes, METEOROLOGY FOR PILOTS, Civil Aeronautics Bulletin No. 26, Sept. 1940, Pp. 158. Price $.30.

E. Richard Heineman, PLANE TRIGONOMETRY, McGraw-Hill Book Company, 1942. Pp. 167. Price $2.00.

Maurice C. Holmes, AN OUTLINE OF PROBABILITY AND ITS USES, Burgess Publishing Company, 1942. Pp. 119. Price $1.50.

A. Hooper, A MATHEMATICS REFRESHER, Henry Holt & Company, 1942. Pp. 342. Price $1.90.

Glenn James and Robert C. James, MATHEMATICS DICTIONARY, The Digest Press, Van Nuys, California, 1942. Pp. 279. Price $3.00.

Edward Jordan, Paul Nelson, William Osterbrock, Fred Pumphrey, Lynne Smeby, edited by W. L. Everitt, FUNDAMENTALS OF RADIO, Prentice-Hall, 1942. Pp. 400. Trade price $5.00. School price $3.75.

Harry M. Keal and Clarence J. Leonard, ESSENTIAL MATHEMATICS FOR SKILLED WORKERS, John Wiley & Sons, Inc., 1942. Pp. 293. Price $1.50.

Lyman Kells, Willis Kern and James Bland, SPHERICAL TRIGONOMETRY WITH NAVAL AND MILITARY APPLICATIONS. McGraw-Hill Book Company, 1942. Pp. 163.

Morris S. Knebelman and Tracy Y. Thomas, PRINCIPLES OF COLLEGE ALGEBRA, Prentice-Hall, 1942. Pp. 380. Price $2.50.

Franklin Wesley Kokomoor, MATHEMATICS IN HUMAN AFFAIRS, Prentice-Hall, 1942. Pp. 754. Price $4.00. Trade price $5.35.

Maurice Kraitchik, MATHEMATICAL RECREATIONS, W. W. Norton & Company, 1942. Pp. 327.

N. J. Lennes, A FIRST COURSE IN ALGEBRA, Revised Edition, The Macmillan Company, 1942. Pp. 562. Price $1.56.

Thoburn C. Lyon, PRACTICAL AIR NAVIGATION, Civil Aeronautics Bulletin No. 24, Sept. 1940. Pp. 245. Price $1.00.

Justin H. Moore, and Julio A. Mira, THE GIST OF MATHEMATICS, Prentice-Hall, 1942. Pp. 726. Price $4.00.

Max Morris and Orley Brown, DIFFERENTIAL EQUATIONS, Prentice-Hall, 1942. Pp. 355. Price $3.00.

Carl Wallace Munshower and James Fletcher Wardwell, BASIC COLLEGE MATHEMATICS, Henry Holt & Company, 1942. Pp. 612. Price $3.20.

National Society for the Study of Education, 41st Yearbook, Part I, PHILOSOPHIES OF EDUCATION, Public School Publishing Co., 1942. Pp. 321.

National Society for the Study of Education, 41st Yearbook, Part II, THE PSYCHOLOGY OF LEARNING, Publich School Publishing Co., 1942. Pp. 502.

Alfred L. Nelson, Karl W. Folley and William M. Borgman, CALCULUS, D. C. Heath & Company, 1942. Pp. 356. Price $2.75.

George Osteyee, MATHEMATICS IN AVIATION, The Macmillan Company, 1942. Pp. 186.

Frank A. Rickey and J. P. Cole, PLANE AND SPHERICAL TRIGONOMETRY, The Dryden Press, 1942. Pp. 209. Price $2.25.

Elizabeth Roudebush, LABORATORY GEOMETRY, Prentice-Hall, 1942. Pp. 192. Price $1.12.

William W. Rogers and Paul L. Welton, BLUEPRINT READING AT WORK, Silver Burdett Company, 1942. Pp. 136. Price $1.28.

G. M. Ruch, F. B. Knight and J. W. Studebaker, ARITHMETIC FOR THE EMERGENCY, Scott, Foresman & Company, 1942. Pp. 176.

William L. Schaaf, MATHEMATICS FOR MECHANICS, Garden City Publishing Company, 1942. Pp. 298. Price $2.00.

The Science Service, YOUTH LOOKS AT SCIENCE AND WAR, Penguin Books, Inc., 1942. Pp. 133. Price $.25.

G. E. F. Sherwood and Angus E. Taylor, CALCULUS, Prentice-Hall, 1942. Pp. 496. Price $3.75.

Daniel T. Sigley and William T. Stratton, SOLID GEOMETRY AND MENSURATION, The Dryden Press, 1942. Pp. 160. Price $1.60.

Edwin R. Smith (General Editor), MILITARY AND NAVAL MAPS AND GRIDS, The Dryden Press, 1942. Pp. 96. Price $1.00.

Atherton H. Sprague, ESSENTIALS OF PLANE AND SPHERICAL TRIGONOMETRY, Prentice-Hall, 1942. Pp. 165. Price $1.35.

Edwin I. Stein, ALGEBRA IN EASY STEPS, Newson & Company, 1942. Pp. 238.

Arthur E. Traxler, THE USE OF TESTS AND RATING DEVICES IN THE APPRAISAL OF PERSONALITY, Educational Records Bureau, 1942. Pp. 75.

E. P. Van Leuven, GENERAL TRADE MATHEMATICS, McGraw-Hill Book Company, 1942. Pp. 575.

Abram Van Zyl, MATHEMATICS AT THE CROSS-ROADS, Maskew Miller, Ltd., Cape Town, South Africa, 1942. Pp. 239 (Dissertation).

William R. Wadden and Lt. Commander James G. Willett, AVIATION MATHEMATICS,

Houghton Mifflin Company, 1942. Pp. 136. Price $.88.

Paul L. Welton and William W. Rogers, SHOP MATHEMATICS AT WORK, Silver Burdett Company, 1942. Pp. 204. Price $1.56.

John H. Wolfe, Wm. F. Mueller and Seibert D. Mullikin, SIMPLIFIED INDUSTRIAL MATHEMATICS, McGraw-Hill Book Company, 1942. Pp. 281. Price $2.00.

Hugh H. Wolfenden, THE FUNDAMENTAL PRINCIPLES OF MATHEMATICAL STATISTICS, The Actuarial Society of America, The Macmillan Company of Canada, Ltd., Toronto, 1942. Pp. 379.

F. Lynwood Wren, FUNCTIONAL NUMBERS, D. C. Heath & Company, 1942. Pp. 316. Price $1.00.

F. Lynwood Wren, NUMBER RELATIONS, D. C. Heath & Company, 1942. Pp. 300. Price $.96.

Avrum Zier, AERODYNAMICS FOR MODEL AIRCRAFT, Dodd, Mead and Company, 1942. Pp. 259. Price $3.00.

The National Council of Teachers of Mathematics

EDWIN W. SCHREIBER, *Secretary*
Western Illinois State Teachers College, Macomb, Illinois

OFFICIAL BALLOT, FEB. 27, 1943

For Second Vice-President, 1943–1944. (Vote for One)

( ) Hesse, Emma, Oakland, Calif.
( ) Olds, Edwin G., Pittsburgh, Pa.

For Board of Directors, 1943–1945. (Vote for Three)

( ) Beck, Hildegarde, Detroit, Mich.
( ) Carnahan, Walter, Indianapolis, Ind.
( ) Charlesworth, H. W., Denver, Colo.
( ) Karnes, Houston T., University, La.
( ) Krathwohl, Wm. C., Chicago, Ill.
( ) Schult, Veryl, Washington, D. C.

Members will please mark this ballot, enclose same in an envelope (3 cents postage) and mail to the above address. Kindly give YOUR name and address on the outside of envelope. Your ballot must reach Macomb by Feb. 27, 1943.